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Published in IET Communications
Received on 28th February 2013
Revised on 3rd July 2013
Accepted on 6th September 2013
doi: 10.1049/iet-com.2013.0167

ISSN 1751-8628

Cooperative multicasting based on superposition and layered coding

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Abstract: Cooperative diversity plays an important role in combating channel fading and increasing reliability of wireless communication links. The main purpose of cooperative diversity is to transmit the same data from multiple sources. Hence, there is no inherent capability in the cooperative diversity schemes to deal with scalable types of data, for example, scalable video coded signals. The authors introduce a two-phase cooperative multicast scheme based on superposition coding to transmit scalable video signals. The new scheme mixes the superposition with cooperative diversity, and chooses the right parameters in both schemes to enhance the system's multicast capability. This study also derives an exact closed-form expression of the average multicast group throughput in case of Rayleigh flat fading channel. The closed-form expression allows system designers to choose the correct cooperation and superposition parameters to satisfy the network operator needs. Simulations show that, in addition to the additional degrees of freedom resulting from using cooperation with superposition, the proposed scheme outperforms the conservative scheme and schemes solely exploiting cooperative relaying or superposition. Simulations show that the new scheme can increase the average network throughput more than four times compared to the conservative scheme.

1 Introduction

Multimedia broadcast and multicast have attracted great attention in the last few years. Various multimedia services have recently gained great popularity, such as Internet Protocol TV (IPTV) and mobile TV, increasing the importance of efficient transmission algorithms that utilise the system resources to serve the subscribers [1–5].

Multicast transmission is necessary in multimedia services, where subscribers that demand the same service are grouped logically to form a multicast group. This is necessary, for example, when multiple users in a cell request to watch the same IPTV channel. The diverse channel conditions of multiple subscribers in the multicast group make it a challenge to adapt the transmission rate to simultaneously satisfy all group members and improve the total group throughput.

A simple multicast scheme has been developed for CDMA 2000 1xEV-DO networks by taking a default transmission rate ignoring the diverse channel conditions among different subscribers [6]. A conservative multicast scheme uses the transmission rate accommodating the worst channel conditions in the group excluding the subscribers that suffer from total fades. As a result, all the subscribers in the multicast group get the service at the same, low quality, level.

Multicast systems are mainly used for video transmission. Many advanced scalable video codecs have been proposed to improve the scalability of video transmission. These codecs enable partial decoding, thus, the video quality increases with the number of quality layers that the receiver decodes correctly. In [7], the authors proposed two-level superposition coded multicasting (SPCM) scheme for IPTV. A multicast signal is generated by superimposing the base quality layer bit stream, modulated with a low-order modulation scheme to the enhancement quality layer. The base layer contains essential information to decode the video stream while the enhancement layer contains information to enhance the quality of the video. Hence, subscribers with bad channel conditions can decode base layer only, while other subscribers decode both base and enhancement layers to obtain better video quality.

In addition to service scalability, delivering multimedia service to subscribers experiencing bad channel conditions is a great challenge in multicast transmission. Recently, cooperative transmission has been a subject of great interest among the research community. It is considered as a desirable enhancement to future systems and is being evaluated for the 4G standards long-term evolution-advanced (LTE-advanced) [8, 9] and WiMAX [10]. Subscribers in a wireless network help each other by forwarding data, aiming at increasing each subscriber's capacity and the aggregate multicast group capacity. This increases the spatial diversity than using fixed relays in cooperative transmission as in [11]. Decode and forward (DF) protocol is widely used in cooperative transmission [12]. Such a scheme divides the downlink time slot into two phases. In the first phase, the base station (BS) broadcasts a message to all subscribers in the multicast group. In the second phase, relays that correctly decoded the message forward it to the subscribers that failed to decode the message during the first phase. In [13], a cognitive radio-assisted cooperation framework is proposed for the downlink transmissions in OFDMA-based cellular network, where relay stations leverage cognitive radio technique to occupy white space sub-channels for relaying transmissions to cellular users. In [14], a fixed number of relays take turns to forward packets, and layered video coding is used to provide subscribers with different video quality. The authors in [15] proposed cooperative multicast scheme (CSM) where all subscribers that correctly decode the message transmitted by the BS in the first phase serve as relays and simultaneously forward the packets in second phase. The work in [16] used a maximal ratio combiner to enhance the received signal-to-noise ratio (SNR) in a distributed CSM.

Utilising superposition and cooperative relaying for unicast transmission is investigated in [11, 17–21] to enhance average network throughput. Although several strategies have been proposed for multicast transmission, there are pitfalls that need to be investigated. For example, subscribers with good channel conditions are deprived from achieving higher throughput owing to their employment in cooperative relaying in the second phase and are not given the opportunity to increase their throughput as in the scheme proposed in [15, 22].

In this paper, we propose a scheme that utilises cooperative transmission with layered video coding and superposition to enable an efficient video multicast transmission. The scheme exploits the spatial diversity gain across multiple subscribers by using two-phase cooperative transmission with superposition coding. In the first phase, the BS broadcasts a composite message of a base and first enhancement layer. The base message will be decoded by most of the subscribers in the multicast group, whereas the first enhancement message will only be decoded by the subscribers that experience better channel conditions. In the second phase, based on the subscribers' SNR, a fraction of the subscribers cooperatively broadcast the first enhancement message. The remaining subscribers that successfully received the first enhancement message will not participate in the cooperative transmission and will receive a second enhancement message that is transmitted by the BS in the second phase.

The scheme allows subscribers with the best channel conditions to obtain the multimedia service at the highest data rates and enhances the rate of subscribers that experience low SNR. Different than the scheme used in [22], subscribers who receive base layer are given opportunity to receive enhancement layer in second phase. Compared to CMS, SPCM and the conservative scheme, the proposed scheme is shown to achieve higher average multicast throughput under the same total energy consumption. The proposed scheme enhances scalable multimedia delivery by providing different rates in the same transmission slot. As an extension to our work in [23], a general-form expression for the average multicast throughput is obtained that can be applied to different multicast schemes for analysis and optimisation. The general expression allows manipulating superposition and cooperation parameters to operate at different points in the total throughput-fairness space.

The rest of the paper is organised as follows: Section 2 reviews the system model for multicasting schemes based on direct transmission, superposition and cooperative transmission. Section 3 introduces the proposed cooperative superposition relaying scheme. In Section 4, we analyse the achievable rates of the proposed scheme and derive a closed-form formulation for the average multicast group throughput, while Section 5 presents the simulation results to demonstrate the effectiveness of the proposed scheme. Finally, Section 6 concludes the paper.

2 System model and related work

A wireless network with one cell of radius R is considered. The BS is located at the centre of the cell and M subscribers are randomly distributed in the cell with a distance r and orientation θ relative to the centre as illustrated in Fig. 1. Signals from the BS to the subscribers are subject to path loss and Rayleigh fading. h_i is the Rayleigh fading channel gain between the BS and subscriber i modelled as a zero-mean circularly symmetric complex Gaussian random variable with unit variance, $h_i \sim \mathrm{CN}(0,\ 1)$. The channels for different subscribers are assumed to be independent and identically distributed (i.i.d.). Perfect channel estimation is assumed at all receivers. All transmitting nodes are perfectly synchronised and the delay spread of the channel is negligible, which is a valid assumption for narrow band wireless communications [24]. The received signal for a given receiver i is affected by additive white Gaussian noise (AWGN) n_i with variance N_0, $n_i \sim \mathrm{CN}(0, N_0)$.

A variety of multicast schemes have been proposed in the literature exploiting fundamental principles such as direct transmission, superposition and cooperative relaying. These fundamental schemes are overviewed in the next subsections.

2.1 Conservative multicast scheme

In the conservative scheme, the BS broadcasts the multimedia stream to all subscribers in the multicast group with a fixed rate R_v. The transmission rate is selected to satisfy subscribers experiencing worst channel conditions, such that the rate is higher than certain threshold. Accordingly, the transmission rate is controlled with the subscriber experiencing the worst channel conditions, but not total failure of the communication link, resulting in

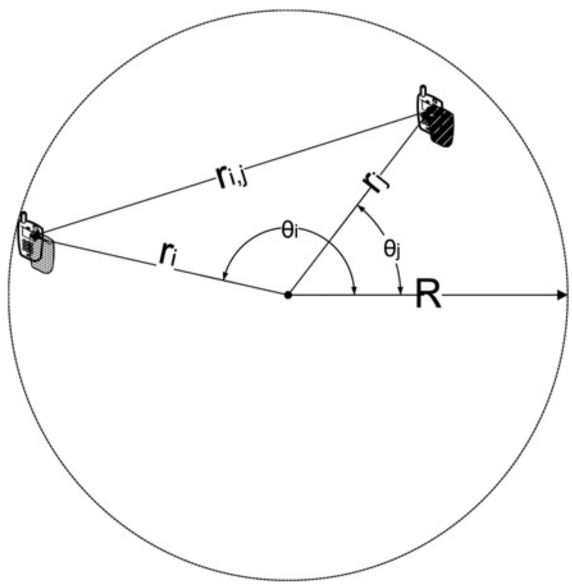

Fig. 1 *System model: two subscribers case*

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doi: 10.1049/iet-com.2013.0167

underutilisation for group subscribers with good channel conditions. The received signal for a given subscriber i in the multicast group can be given by

$$y_i^v = h_i\sqrt{P_d r_i^{-n}}x^v + n_i \quad (1)$$

where x^v is the transmitted signal by the BS after encoding the multimedia service into a single stream and y_i^v is the received signal by subscriber i. The superscript v stands for conservative multicast scheme. P_d is the transmitting signal power and n is the path loss exponent. The model used assumes that $P_R \propto (P_T/r_i^n)$, where P_R and P_T are the received and transmitted powers, respectively.

According to (1), the received SNR for the subscriber i under conservative scheme transmission is given by $\text{SNR}_i^v = (|h_i|^2 P_d r_i^{-n}/N_0)$.

2.2 Multicast scheme based on superposition

Utilising superposition in multicast transmission of multimedia services was first introduced by [7]. Using layered video coding, the multimedia stream can be coded into two streams allowing partial decoding at the receiver at two different layers. The BS broadcasts a composite message that consists of base layer and enhancement layer with two different rates. If the total transmitting power is P_d with power allocation fraction a, the power aP_d and $\bar{a}P_d$ are allocated to the enhancement and base messages, respectively, where $\bar{a} = 1 - a$. The transmitting rates depend mainly on the power allocation factor a [25]. Subscribers with better channel conditions manage to decode both the base and enhancement layers resulting in better service quality. On the other hand, subscribers with bad channel conditions decode only the base layer, resulting in successfully decoding of the video, but with a worse video quality.

The received signal for a given subscriber i in the multicast group can be given by

$$y_i^s = h_i\sqrt{aP_d r_i^{-n}}x_b^s + h_i\sqrt{\bar{a}P_d r_i^{-n}}x_e^s + n_i \quad (2)$$

where x_b^s is the base layer message, x_e^s is the enhancement layer message, y_i^s is the signal received by subscriber i and the superscript 's' stands for superposition scheme. The effective received SNR for the base message can be given by $\text{SNR}_{i,b}^s = ((|h_i|^2\bar{a}P_d r_i^{-n})/(|h_i|^2 aP_d r_i^{-n} + N_0))$, where the enhancement message is considered as an interference on the base message [26]. In case that subscriber i managed to decode the base message correctly, the base message will be subtracted from the received signal. In this case, the effective received SNR for the enhancement message is given by $\text{SNR}_{i,e}^s = (|h_i|^2 aP_d r_i^{-n}/N_0)$.

2.3 Simple cooperative scheme

In cooperative multicast, the downlink time is divided into two phases. In the first phase, the BS broadcasts a message to all subscribers in the multicast group. In the second phase, subscribers that managed to decode the message cooperatively retransmit the message to other subscribers in the multicast group. The received signal for a given subscriber i in the multicast group can be given by

$$y_i^{c,1} = h_i\sqrt{P_d r_i^{-n}}x^{c,1} + n_i \quad (3)$$

where $x^{c,1}$ is the message transmitted by the BS in the first phase and $y_i^{c,1}$ is the signal received by subscriber i. The superscript c, 1 stands for first phase in cooperative scheme. The SNR of subscriber i in the first phase is given by $\text{SNR}_i^{c,1} = (|h_i|^2 P_d r_i^{-n}/N_0)$. Only N out of M subscribers correctly decode the message transmitted in the first phase. These N subscribers work as relays and cooperatively retransmit the message in the second phase. Therefore, for a subscriber j that failed to decode the message in the first phase, the received signal in the second phase is given by [24]

$$y_j^{c,2} = \sum_{i=1}^{N} h_{i,j}\sqrt{r_{i,j}^{-n}P_{dr}}x^{c,2} + n_i \quad (4)$$

$h_{i,j}$ is the Rayleigh fading channel gain between the subscribers i and j modelled as a zero-mean circularly symmetric complex Gaussian random variable with unit variance, $h_{i,j} \sim \text{CN}(0, 1)$. P_{dr} is the relay transmitting power. $r_{i,j}$ is the relative distance between the subscribers i and j, where

$$r_{i,j}^2 = r_i^2 + r_j^2 - 2r_i r_j \cos(\theta_i - \theta_j)$$

$x^{c,2}$ is the message transmitted by the subscriber i in the second phase and $y_j^{c,2}$ is the signal received by subscriber j. The superscript c, 2 stands for second phase in cooperative scheme. The received SNR for subscriber j in the second phase, according to (4), is given by

$$\text{SNR}_j^{c,2} = \left(\left(\left|\sum_{i=1}^{N} h_{i,j}\sqrt{r_{i,j}^{-n}P_{dr}}\right|^2\right)\Big/N_0\right)$$

For fair comparison, we limit total transmission power used by relays in cooperative relaying to the total transmitted power used in conservative multicast and multicast based on superposition schemes in the second phase. Thus, $P_d = NP_{dr}$.

3 Proposed scheme

The objectives of the proposed scheme are to increase the average multicast group throughput, deliver the service to subscribers experiencing bad channel conditions and enable scalable delivery of multimedia data, depending on the average channel conditions of different users. Fig. 2 illustrates the principle of the proposed CSM. In this scheme, the downlink frame that has a total duration T is divided into two phases of equal durations T_1 and T_2. The multimedia transmission in the two phases is discussed as follows.

3.1 First phase

Using layered coding, the multimedia stream can be coded into multiple streams allowing partial decoding at the receiver at different layers. An example for superposition modulation is illustrated in Fig. 3. The BS broadcasts a signal which is a composite message of base layer B_1 and first enhancement layer E_1 with rates R_{B1} and R_{E1}, respectively.

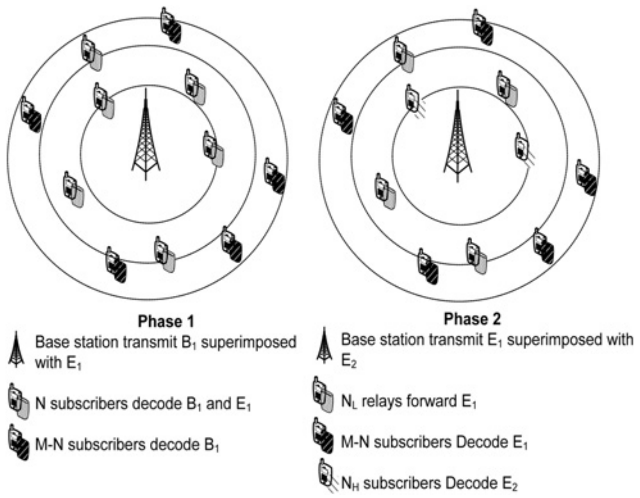

Fig. 2 *Proposed multicast cooperative scheme*

For a given subscriber i in the multicast group, the received signal in the first phase is given by

$$y_i^{\text{sc},1} = h_i\sqrt{\bar{a}P_d r_i^{-n}}x_{\text{b}}^{\text{sc},1} + h_i\sqrt{aP_d r_i^{-n}}x_{\text{e}}^{\text{sc},1} + n_i \quad (5)$$

where $x_{\text{b}}^{\text{sc},1}$ and $x_{\text{e}}^{\text{sc},1}$ are the base and enhancement layer messages, respectively, in the first phase, $y_i^{\text{sc},1}$ is the received signal by subscriber i and the superscript sc, 1 stands for first phase in superposition and cooperative scheme. For the subscribers that manage to decode B_1 correctly, the effective received SNR for the base layer message is given by $\text{SNR}_{\text{b},i}^{\text{sc},1} = \left(\left(|h_i|^2\bar{a}P_d r_i^{-n}\right)/\left(|h_i|^2 aP_d r_i^{-n} + N_0\right)\right)$ where the enhancement message is considered as interference on the base layer message. In case that the subscriber i managed to decode B_1 correctly, the receiver will subtract the base layer message from the received signal. The effective received SNR of the enhancement message after subtraction is given by $\text{SNR}_{\text{e}1,i}^{\text{sc},1} = \left(|h_i|^2 aP_d r_i^{-n}/N_0\right)$. N out of M subscribers, with good channel conditions, can decode both B_1 and E_1 with negligible probability of error. Let the set S_N has the N subscribers that can decode both B_1 and E_1 messages, resulting in a better quality of the multimedia service.

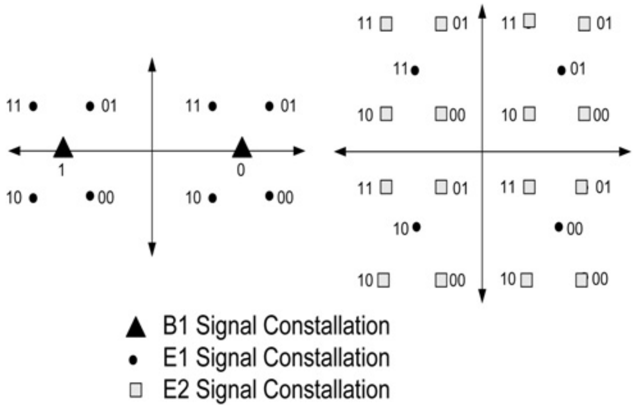

Fig. 3 *Signal constellation for modulation used in cooperative scheme*

3.2 Second phase

The subscribers in set S_N are subdivided into two sets S_{NH} and S_{NL}, depending on the received SNR in the first phase. The sets S_{NH} and S_{NL} have N_H and N_L subscribers, respectively. Therefore, in the second phase subscribers are divided into three sets:

1. S_{NH} has the N_H subscribers that successfully decoded E_1 in the first phase and $\text{SNR}_{\text{e}2,i}^{\text{sc},1}$, $i \in S_{NH}$, is higher than a certain threshold $\gamma_{\text{e}}^2 = 2^{R_{E2}} - 1$, where R_{E2} is the rate of the second enhancement layer E_2.
2. S_{NL} has the N_L subscribers that successfully decoded E_1 in the first phase and $\text{SNR}_{\text{e}2,i}^{\text{sc},1}$, $i \in S_{NL}$, is lower than γ_{e}^2.
3. S_{M-N} has the $M - N$ subscribers that failed to decode E_1 in the first phase, where $N = N_H + N_L$. Some of these subscribers managed to decode B_1 correctly.

In the second phase of the downlink, the procedure goes as follows:

- Subscribers belonging to S_{NL} serve as relays and cooperatively transmit E_1 with rate R_{E1}.
- The BS broadcasts E_2 with rate R_{E2}.
- Subscribers belonging to S_{M-N} decode the message E_1 that is transmitted by the relays in set S_{NL}. Only subscribers that managed to decode B_1 will benefit from successful reception of E_1 in the second phase.
- Subscribers belonging to S_{NH} decode the message E_2 after cancelling the effect of the interference caused by the cooperative relays.

In the first phase, subscribers in the set S_{M-N} fail to decode E_1 due to bad channel conditions with the BS. In the second phase, cooperative transmission enhances the rates of the set S_{M-N}. The enhancement is due to exploiting the spatial diversity provided by the relaying subscribers in the cell. For a given subscriber $k \in S_{M-N}$, the received signal in the second phase is given by

$$y_k^{\text{sc},2} = \sum_{i=1}^{N_L} h_{i,k}\sqrt{r_{i,k}^{-n}P_{\text{dr}}}x_{1\text{e}}^{\text{sc},2} + h_k\sqrt{aP_d r_k^{-n}}x_{2\text{e}}^{\text{sc},2} + n_k \quad (6)$$

where $x_{1\text{e}}^{\text{sc},2}$ is the signal transmitted cooperatively by N_L subscribers in the second phase. The subscript 1e stands for first enhancement layer and the superscript sc,2 stands for second phase superposition and cooperative scheme. $x_{2\text{e}}^{\text{sc},2}$ is the embedded signal transmitted by the BS in the second phase and the subscript 2e stands for second enhancement layer. $r_{i,k}$ is the distance between subscriber i and subscriber j and is given by

$$r_{i,k}^2 = r_i^2 + r_k^2 - 2r_k r_i \cos\left(\theta_i - \theta_j\right)$$

The effective received SNR for subscriber k in the second phase is given by

$$\text{SNR}_{\text{e}_1,k}^{\text{sc},2} = \frac{\left|\sum_{i=1}^{N_L} h_{i,k}\sqrt{r_{i,k}^{-n}P_{\text{dr}}}\right|^2}{|h_k|^2 aP_d r_k^{-n} + N_0}$$

where the embedded signal transmitted by the BS is considered as interference on the received signal. For fair comparison, we select total transmitted power by

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doi: 10.1049/iet-com.2013.0167

cooperative relays and the BS in the second phase to be equal to total transmitted power used in transmission in the multicast schemes based on direct transmission, superposition and simple cooperative transmission. Thus, $\bar{a}P_d = N_L P_{\text{dr}}$.

A key element in the proposed procedure that should be noted is that the interfering signal received power caused by the BS transmission of E_2 is low compared to E_1 received power. This is due to the already bad channel conditions on the direct link between the BS and the subscribers in the set S_{M-N}. This is evident since all the subscribers in the set S_{M-N} failed to decode E_1 transmitted by the BS with the same power correctly in the first phase. Therefore, the signal power of E_2 should be limited and not to exceed the signal power of E_1 in the second phase.

Before the N_H subscribers decode E_2, they eliminate the interference effect of the cooperative relays transmission due to the prior knowledge of E_1 and the channels gains to the other subscribers. In terms of channel estimation, the technique described in [27] is employed to reduce computational complexity. This technique does not require the receiver to estimate the channel gains with each transmitter. Accordingly, the effective received SNR for a given subscriber $j \in S_{NH}$ is given by $\text{SNR}^{\text{sc},2}_{e_2,j} = \left(|h_j|^2 aP_d r_j^{-n}/N_0\right)$.

To decrease processing overhead on the BS, all rates will be determined based on long-term channel conditions as in [15]. The scheme enables scalable multimedia delivery by providing three different rates for three sets of subscribers. The scheme exploits the capability of the subscribers experiencing good channel conditions to receive service at high rates and high quality of service.

Choosing the rates R_{B1}, R_{E1} and R_{E2} is critical for the system performance. These rates determine the number of subscribers in the three sets stated above. Accordingly, the system can be driven to enhance the throughput of the subscribers experiencing good channel conditions by increasing the number of subscribers in the set S_{NH} which increases the number of receivers in the second phase on the expense of enhancing throughput of subscribers in the set S_{M-N}. On the other hand, the system can be driven to enhance the throughput of subscribers experiencing bad channel conditions by increasing the number of cooperative relays in the set S_{NL} on the expense of increasing the rate of subscribers that are able to receive a second enhancement layer. Since E_1 is transmitted in the first and second phase with the same rate R_{E1}, therefore T_1 is equal to T_2 to ensure that $T_1R_{E1} = T_2R_{E1}$.

For controlling the system thresholds, used to determine N_H and N_L, two parameters are introduced: coverage ratio C and cooperative ratio α. The coverage ratio is the percentage of subscribers, on average, that can correctly receive the first enhancement layer rate R_{E1} in the first phase and is given by $C = E(N)/M$. The cooperative ratio is the percentage of subscribers, on average, that work as relays in the second phase and is given by $\alpha = E(N_L)/M$.

In the first phase, C is used to determine R_{E1}. As C increases, R_{E1} decreases to enable more subscribers to receive the enhancement layer, that is, N increases. As C decreases, R_{E1} increases and a lower number of subscribers will be able to decode the enhancement layer correctly in the first phase.

In the second phase, α is used to determine R_{E2}. As α increases more subscribers cooperate in transmission in the second phase, that is, N_L increases. Increasing N_L improves the probability that the subscribers in set S_{M-N} decode E_1 correctly in the second phase. Since the set S_{NH} has the subscribers with the highest SNR from the set S_N, increasing α decreases N_H resulting in higher R_{E2} and vice versa.

The parameter α controls the system by either favouring subscribers with high received SNR on the expense of subscribers with low SNR, or enhancing the system fairness by increasing the number of cooperative subscribers N_L which leads to higher throughput for subscribers with low SNR.

4 Performance analysis

We first derive an expression of average multicast throughput for conservative and superposition schemes that will be exploited in the derivation of the exact closed-form expression of the average multicast group throughput for the proposed scheme. Average throughput is defined as the probability of successful decoding of messages at a certain rate R, multiplied by R. For the Rayleigh flat fading channel, given a threshold γ, the maximum achievable data rate with a negligible error probability is $\log_2(1+\gamma)$ for unit bandwidth [28]. Therefore, if the received SNR is higher than γ, the subscriber is assumed to decode the message with negligible probability of error and the throughput achieved is the probability of correct decoding multiplied by rate of transmission. We assume that for a given subscriber, the joint probability density function of the distance between the subscriber and the BS r and the angle θ is given by $f(r, \theta) = r^2/\pi R^2$ with $0 \le r \le R$ and $0 \le \theta \le 2\pi$. The marginal distribution of r is given by $f(r) = 2r/R^2$ and θ is uniformly distributed over the range $[0, 2\pi]$ [24].

4.1 Conservative multicast scheme

In conservative scheme, the BS broadcasts the multimedia stream with a conservative rate R_v. For a given subscriber i located at a distance r_i from the BS, the received SNR_i^v follows an exponential distribution with a parameter $\lambda = \left(N_0 r_i^{-n}/P_d\right)$ under the assumption of Rayleigh fading channel. The average multicast group throughput for conservative scheme can be given by [24]

$$\begin{aligned} R &= \frac{R_v \int \ldots \int \sum_{i=1}^{M} P\left(\text{SNR}_i^v > \gamma_v/r_i\right) f(r_1)\ldots f(r_M)\,\mathrm{d}r_1 \ldots \mathrm{d}r_M}{M} \\ &= R_v \int_0^R P\left(\text{SNR}_i^v > \gamma_v/r_i\right) f(r_i)\,\mathrm{d}r_i \\ &= R_v \int_0^R \exp\left(\frac{-N_0 r_i^{-n}\gamma_v}{P_d}\right)\frac{2r_i}{R^2}\,\mathrm{d}r_i \\ &= \frac{2R_v}{nR^2}\left(\frac{P_d}{N_0\gamma_v}\right)^{(2/n)}\Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_v}{P_d}\right) \end{aligned} \tag{7}$$

where $\gamma_v = 2^{R_v} - 1$ and $\Gamma(a, x) = \int_0^x exp^{-t} t^{a-1}\,\mathrm{d}t$ is the incomplete Gamma function. The second equality in (7) is due to independent and identical distribution of user locations.

4.2 Multicast scheme based on superposition

In the multicast scheme based on superposition, the BS broadcasts the base and enhancement messages with rates $R_{\rm b}^{\rm s}$ and $R_{\rm e}^{\rm s}$, respectively. For a given subscriber i located at a distance r_i from the BS, the received SNR for the base layer message is given by $\text{SNR}_{i,\rm b}^{\rm s} = \left(\left(|h_i|^2 \bar{a} r_i^{-n} P_d/N_0\right)/\left(|h_i|^2 a r_i^{-n} P_d/N_0 + 1\right)\right)$. Let $x = |h_i|^2 r_i^{-n} P_d/N_0$ where x follows an exponential distribution with a parameter $\lambda_{\rm b} = \left(N_0 r^{-n}/P_d\right)$. Therefore, $\text{SNR}_{i,\rm b}^{\rm s} = \left(\bar{a}x/ax + 1\right)$. Using Jacobian transformation to obtain the probability density function of $\text{SNR}_{i,\rm b}^{\rm s}$, it can be shown that

$$
\begin{aligned}
R &= \frac{R_v \int \ldots \int \sum_{i=1}^{M} P\left(\text{SNR}_i^v > \gamma_v / r_i\right) f(r_1) \ldots f(r_M) \mathrm{d}r_1 \ldots \mathrm{d}r_M}{M} \\
&= R_v \int_0^R P\left(\text{SNR}_i^v > \gamma_v / r_i\right) f(r_i) \mathrm{d}r_i \\
&= R_v \int_0^R \exp\left(\frac{-N_0 r_i^{-n} \gamma_v}{P_d}\right) \frac{2r_i}{R^2} \mathrm{d}r_i \\
&= \frac{2R_v}{nR^2} \left(\frac{P_d}{N_0 \gamma_v}\right)^{(2/n)} \Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_v}{P_d}\right)
\end{aligned}
$$

The probability of correct decoding of the base message given user location is given by

$$
\begin{aligned}
p\left(\text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}/r_i\right) &= \int_{\gamma_b}^{\infty} p\left(\text{SNR}_{i,\rm b}^{\rm s}/r_i\right) \mathrm{dSNR}_{i,\rm b}^{\rm s} \\
&= \int_{\gamma_b}^{(\bar{a}/a)} \frac{\lambda_{\rm b} \bar{a}}{\left(\bar{a} - a\text{SNR}_{i,\rm b}^{\rm s}\right)^2} \\
&\quad \times \exp\left(\frac{-\lambda_b \text{SNR}_{i,\rm b}^{\rm s}}{\bar{a} - a\text{SNR}_{i,\rm b}^{\rm s}}\right) \mathrm{dSNR}_{i,\rm b}^{\rm s} \\
&= \exp\left(\frac{-\lambda_{\rm b} \gamma_{\rm b}}{\bar{a} - a\gamma_{\rm b}}\right)
\end{aligned} \tag{8}
$$

where $\gamma_{\rm b} = 2^{R_{\rm b}^{\rm s}} - 1$. If subscriber i managed to decode the base message, it will subtract the base message signal from the received signal. The effective received SNR for the enhancement layer given subscriber location follows exponential distribution with a parameter $\lambda_a = \left(N_0 r_i^{-n}/aP_d\right)$.

The enhancement message for a video stream is useful only in case that base message is correctly decoded. This is due to the fact that enhancement message contains information for better video quality but the information essential for stream decoding are in base message. The probability of correct decoding of enhancement message and base message given user location is given by

$$
\begin{aligned}
& p\left(\text{SNR}_{i,\rm e}^{\rm s} > \gamma_{\rm e}, \text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}/r_i\right) \\
&= p\left(|h_i|^2 a r_i^{-n} P_d/N_0 > \gamma_{\rm e}, \frac{|h_i|^2 \bar{a} r_i^{-n} P_d/N_0}{|h_i|^2 a r_i^{-n} P_d/N_0 + 1} > \gamma_{\rm b}/r_i\right) \\
&= p\left(|h_i|^2 r_i^{-n} P_d/N_0 > \frac{\gamma_{\rm e}}{a},\right. \\
&\qquad \left. |h_i|^2 r_i^{-n} P_d/N_0 > \frac{\gamma_{\rm b}}{(1-a) - a\gamma_{\rm b}}/r_i\right) \\
&= p\left(|h_i|^2 r_i^{-n} P_d/N_0 > \frac{\gamma_{\rm e}}{a}/r_i\right)
\end{aligned} \tag{9}
$$

where $\gamma_{\rm e} = 2^{R_{\rm e}^{\rm s}} - 1$. The third equality in (3) is because of

$$
\frac{\gamma_{\rm e}}{a} > \frac{\gamma_{\rm b}}{(1-a) - a\gamma_{\rm b}} \tag{10}
$$

Moreover, the enhancement message is sent with much higher rate than base message rate to serve the subscribers experiencing good channel conditions.

The average multicast group throughput is given by (see (11))

where $\dot{\gamma} = \left(\gamma_{\rm b}/\bar{a} - a\gamma_{\rm b}\right)$. The second equality in (11) is because of independent and identical distribution of subscriber locations.

4.3 Proposed scheme

The proposed scheme utilises superposition and cooperative relaying by transmission over two phases.

1. *First phase:* The analysis of the number of bits transmitted in the first phase is similar to that of superposition scheme over a transmission period T_1.

In the first phase, the BS broadcasts the base and enhancement messages with rates R_B and $R_{E,1}$, respectively. For a given subscriber i located at a distance r_i from the BS, the effective received SNR for the base layer message is given by $\text{SNR}_{i,\rm b}^{\rm sc} = \left(\left(|h_i|^2 \bar{a} P_d/N_0 d_i^{-n}\right)/\left(|h_i|^2 a P_d/N_0 d_i^{-n} + 1\right)\right)$. The probability of correct decoding of the

$$
\begin{aligned}
R &= \frac{\int \ldots \int \sum_{i=1}^{M} \left(R_{\rm b}^{\rm s} P\left(\text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}/r_i\right) + R_{\rm e}^{\rm s} P\left(\text{SNR}_{i,\rm e}^{\rm s} > \gamma_{\rm e}/r_i, \text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}\right)\right) f(r_i) \mathrm{d}r_i}{M} \\
&= \int_0^R \left(R_{\rm b}^{\rm s} P\left(\text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}/r_i\right) + R_{\rm e}^{\rm s} P\left(\text{SNR}_{i,\rm e}^{\rm s} > \gamma_{\rm e}/r_i, \text{SNR}_{i,\rm b}^{\rm s} > \gamma_{\rm b}\right)\right) f(r_i) \mathrm{d}r_i \\
&= \frac{2}{nR^2} \left(\frac{P_d}{N_0}\right)^{(2/n)} \left[R_{\rm b}^{\rm s} \left(\frac{1}{\dot{\gamma}}\right)^{(2/n)} \Gamma\left(\frac{2}{n}, \frac{R^n N_0' \gamma}{P_d}\right) + R_{\rm e}^{\rm s} \left(\frac{a}{\gamma_{\rm e}}\right)^{(2/n)} \Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_{\rm e}}{P_d}\right) \right]
\end{aligned} \tag{11}
$$

doi: 10.1049/iet-com.2013.0167

base message given user location is given by

$$\begin{aligned} p\big(\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) &= \int_{\gamma_\mathrm{b}}^{\infty} p\big(\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}}/r_i\big)\mathrm{dSNR}_{i,\mathrm{b}}^{\mathrm{s}} \\ &= \int_{\gamma_\mathrm{b}}^{(\bar{a}/a)} \frac{\lambda_b \bar{a}}{\big(\bar{a} - a\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}}\big)^2} \exp\left(\frac{-\lambda_b \mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}}}{\bar{a} - a\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}}}\right)\mathrm{dSNR}_{i,\mathrm{b}}^{\mathrm{sc}} \\ &= \exp\left(\frac{-\lambda_\mathrm{b}\gamma_\mathrm{b}}{\bar{a} - a\gamma_\mathrm{b}}\right) \end{aligned} \tag{12}$$

where $\gamma_\mathrm{b} = 2^{R_\mathrm{b}} - 1$. Assuming that subscriber i managed to decode the base message, it will subtract it from the received signal. The effective received SNR for the enhancement layer given subscriber location follows exponential distribution with a parameter $\lambda_a = \big(N_0 r_i^{-n}/aP_d\big)$. The total number of bits transmitted in the first phase is given by (see (13))

where

$$\dot{\gamma} = \frac{\gamma_\mathrm{b}}{\bar{a} - a\gamma_\mathrm{b}} \quad \text{and} \quad \gamma_{\mathrm{e}1} = 2^{R_{E1}} - 1$$

The second equality in (13) is because of independent and identical distribution of subscriber locations.

According to the above analysis, subscribers after first stage decoding are divided into three sets S_{M-N}, S_{N_L} and S_{N_H} with number of subscribers equals to $M-N$, N_L and N_H, respectively, with the following probability

$$\begin{aligned} p_\mathrm{dec} &= p\Big(|S_{M-N}| = M-N,\, |S_{N_L}| = N_L,\, |S_{N_H}| = N_H\Big) \\ &= \prod_{i \in M-N} p\big(\mathrm{SNR}_{i,\mathrm{e}}^{\mathrm{sc}} < \gamma_\mathrm{e}^1,\, \mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) \\ &\quad \times \prod_{k \in N_L} p\big(\gamma_\mathrm{e}^1 < \mathrm{SNR}_{k,\mathrm{e}}^{\mathrm{sc}} < \gamma_\mathrm{e}^2,\, \mathrm{SNR}_{k,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_k\big) \\ &\quad \times \prod_{j \in N_H} p\Big(\mathrm{SNR}_{j,\mathrm{e}}^{\mathrm{sc}} > \gamma_\mathrm{e}^2,\, \mathrm{SNR}_{j,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_j\Big) \end{aligned} \tag{14}$$

2. *Second phase:* For a given decoding result in the first phase, subscribers are divided into three sets S_{NH}, S_{NL} and S_{M-N}. For a given subscriber $i \in S_{M-N}$ at a location (r_i, θ_i), the effective SNR of E_1 signal in the second phase is given by

$$\mathrm{SNR}_{\mathrm{e}_1,i}^{\mathrm{sc},2} = \frac{\left|\sum_{k=1}^{N_L} h_{i,k}\sqrt{r_{i,k}^{-n}P_{dr}}\right|^2}{|h_i|^2 a P_d r_i^{-n} + N_0} \tag{15}$$

In (15), the E_2 received signal power is considered as interference on the reception of E_1. Since h_i is assumed constant over the entire transmission period, and subscriber i failed to decode E_1 in the first phase, therefore $|h_i|^2 a r_i^{-n} P_d/N_0 < \gamma_\mathrm{e}^1$, that is, the interfering signal power does not exceed γ_e^1 and, therefore, $\mathrm{SNR}_{\mathrm{e}_1,i}^{\mathrm{sc},2}$ is dependent on the first phase decoding result.

Let

$$\zeta = \mathrm{SNR}_{\mathrm{e}_1,i}^{\mathrm{sc},2} = \frac{|x|^2}{y+1}$$

where x is a sum of complex normal distributions, therefore

$$x\tilde{\mathrm{CN}}\left(0,\, \sum_{k=1}^{N_L} r_{i,k}^{-n} P_\mathrm{dr}/N_0\right) \quad \text{and} \quad |x|^2$$

follows exponential distribution with a parameter

$$\lambda_1 = \frac{1}{\sum_{k=1}^{N_L} r_{i,k}^{-n} P_\mathrm{dr}/N_0}$$

The probability density function of y given that $y < \gamma_\mathrm{e}^1$ is given by

$$f\big(y/y < \gamma_\mathrm{e}^1\big) = \begin{cases} \dfrac{\lambda_2 \exp(-\lambda_2 y)}{1 - \exp\big(-\lambda_2 \gamma_\mathrm{e}^1\big)}, & 0 < y < \gamma_\mathrm{e}^1 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda_2 = \big(N_0 r_i^{-n}/aP_d\big)$. Using Jacobian transformation to obtain the probability density function of ζ given user location and decoding result in first phase, we obtain

$$f\big(\zeta/r_i,\, \theta_i,\, p_\mathrm{dec}\big) = \frac{\lambda_1\lambda_2 e^{-\zeta\lambda_1}\big(1 - \big(1 + \gamma_\mathrm{e}^1\big)\big)e^{-\gamma_\mathrm{e}^1(\zeta\lambda_1 + \lambda_2)}}{\big(1 - e^{-\lambda_2\gamma_\mathrm{e}^1}\big)} + \frac{\lambda_1\lambda_2 e^{-\zeta\lambda_1}\Big(1 - e^{-\gamma_2(\zeta\lambda_1 + \lambda_2)}\Big)}{\big(1 - e^{-\lambda_2\gamma_\mathrm{e}^1}\big)} \tag{16}$$

Thus given user location and decoding result in first phase, the probability of correct decoding of subscriber $i \in S_{M-N}$ is given by

$$p\big(\zeta > \gamma_\mathrm{e}^1/r_i,\, \theta_i,\, p_\mathrm{dec}\big) = \frac{\lambda_2 e^{-\gamma_\mathrm{e}^1\lambda_1}\Big(1 - e^{-\gamma_\mathrm{e}^1(\lambda_1\gamma_\mathrm{e}^1 + \lambda_2)}\Big)}{\big(1 - e^{-\lambda_2\gamma_\mathrm{e}^1}\big)\big(\lambda_1\gamma_\mathrm{e}^1 + \lambda_2\big)} \tag{17}$$

By averaging (17) for all subscribers $i \in S_{M-N}$ and all possible relaying results in the first stage, we can obtain average number of received bits for the set S_{M-N} given

$$\begin{aligned} b_1 &= \frac{T_1 \int \ldots \int \sum_{i=1}^{M} \big(R_B P\big(\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) + R_{E1} P(\mathrm{SNR}_{i,\mathrm{e}}^{\mathrm{sc}} > \gamma_\mathrm{e},\, \mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) f(r_i)\mathrm{d}r_i}{M} \\ &= T_1 \int_0^R \big(R_B P\big(\mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) + R_{E1} P\big(\mathrm{SNR}_{i,\mathrm{e}}^{\mathrm{sc}} > \gamma_\mathrm{e},\, \mathrm{SNR}_{i,\mathrm{b}}^{\mathrm{sc}} > \gamma_\mathrm{b}/r_i\big) f(r_i)\mathrm{d}r_i \\ &= \frac{2T_1}{nR^2}\left(\frac{P_d}{N_0}\right)^{(2/n)}\left[R_B\left(\frac{1}{\dot{\gamma}}\right)^{(2/n)}\Gamma\left(\frac{2}{n},\, \frac{R^n N_0 \dot{\gamma}}{P_d}\right) + R_{E1}\left(\frac{a}{\gamma_{\mathrm{e}1}}\right)^{(2/n)}\Gamma\left(\frac{2}{n},\, \frac{R^n N_0 \gamma_{\mathrm{e}1}}{P_d}\right)\right] \end{aligned} \tag{13}$$

user locations as follows

$$p_d = \frac{T_2 R_{E1}}{M} \sum_{S_N} \sum_{S_{NL}} \sum_{i \in S_{M-N}} p\big(\zeta_i > \gamma_e^1, \mathrm{SNR}_{e1,i}^{sc} < \gamma_e^1/r_i, \theta_i, p_{dec}\big) p_{dec}$$

$$\begin{aligned}
&= \frac{T_2 R_{E1}}{M} \sum_{S_N} \sum_{S_{NL}} \sum_{i \in S_{M-N}} p\big(\zeta_i > \gamma_e^1, \mathrm{SNR}_{e1,i}^{sc} < \gamma_e^1/r_i, \theta_i, p_{dec}\big) \\
&\quad \times \prod_{l \in M-N} p\big(\mathrm{SNR}_{e1,l}^{sc} < \gamma_e^1, \mathrm{SNR}_{l,b}^{sc} > \gamma_b/r_l\big) \\
&\quad \prod_{k \in N_L} p\big(\gamma_e^1 < \mathrm{SNR}_{e2,k}^{sc} < \gamma_e^2, \mathrm{SNR}_{k,b}^{sc} > \gamma_b/r_k\big) \\
&\quad \times \prod_{j \in N_H} p\Big(\mathrm{SNR}_{e2,j}^{sc} > \gamma_e^2, \mathrm{SNR}_{j,b}^{sc} > \gamma_b/r_j\Big) \\
&= \frac{T_2 R_{E1}}{M} \sum_{N=1}^{M-1} \sum_{|S_N|=N} \sum_{N_L=1}^{N} \sum_{|S_{NL}|=N_L} \sum_{i \in S_{M-N}} p \\
&\quad \times \big(\zeta_i > \gamma_e^1, \mathrm{SNR}_{e1,i}^{sc} < \gamma_e^1/r_i, \theta_i, p_{dec}\big) \\
&\quad \times \prod_{\substack{l \in M-N \\ l \neq i}} p\big(\mathrm{SNR}_{e1,l}^{sc} < \gamma_e^1, SNR_{l,b}^{sc} > \gamma_b/r_l\big) \\
&\quad \prod_{k \in N_L} p\big(\gamma_e^1 < \mathrm{SNR}_{e1,k}^{sc} < \gamma_e^2, \mathrm{SNR}_{k,b}^{sc} > \gamma_b/r_k\big) \\
&\quad \times \prod_{j \in N_H} p\Big(\mathrm{SNR}_{e2,j}^{sc} > \gamma_e^2, \mathrm{SNR}_{j,b}^{sc} > \gamma_b/r_j\Big)
\end{aligned} \tag{18}$$

In (18), the summation excludes $N = 0$ where no subscriber cooperate in relaying and $N = M$ where all subscribers decode the enhancement layer message correctly, then the set S_{M-N} is empty. $|S_{NL}|$ and $|S_{NH}|$ are the number of subscribers in the sets S_{NL} and S_{NH}, respectively. By integrating p_d given by (18) with respect to user locations to obtain b_2^r as

$$\begin{aligned}
b_2^r &= \int \ldots \int p_d f(r_1, \theta_1) \ldots f(r_M, \theta_M) \mathrm{d}r_1 \mathrm{d}\theta_1 \ldots \mathrm{d}r_M \mathrm{d}\theta_M \\
&= \frac{T_2 R_{E1}}{M} \sum_{N=1}^{M-1} \sum_{|S_N|=N} \sum_{N_L=1}^{N} \sum_{|S_{NL}|=N_L} \sum_{i \in S_{M-N}} \prod_{\substack{l \in M-N \\ l \neq i}} \int_0^R \int_0^{2\pi} p \\
&\quad \times \big(\mathrm{SNR}_{e1,l}^{sc} < \gamma_e^1/r_l\big) f\big(r_l, \theta_l\big) \mathrm{d}r_l \mathrm{d}\theta_l \\
&\quad \times \prod_{j \in N_H} \int_0^R \int_0^{2\pi} p\Big(\mathrm{SNR}_{e2,j}^{sc} > \gamma_e^2/r_j\Big) f\Big(r_j, \theta_j\Big) \mathrm{d}r_j \mathrm{d}\theta_j \\
&\quad \times \int \ldots \int p\big(\zeta_i > \gamma_e^1, \mathrm{SNR}_{e1,i}^{sc} < \gamma_e^1, \mathrm{SNR}_{i,b}^{sc} > \gamma_b/r_i, \theta_i, p_{dec}\big) \\
&\quad \times f\big(r_i, \theta_i\big) \times \prod_{k \in N_L} p\big(\gamma_e^1 < \mathrm{SNR}_{e2,k}^{sc} < \gamma_e^2/r_k\big) \\
&\quad \times f(r_k, \theta_k) \mathrm{d}r_1 \mathrm{d}\theta_1 \ldots \mathrm{d}r_{NL} \mathrm{d}\theta_{NL} \mathrm{d}r_i \mathrm{d}\theta_i
\end{aligned} \tag{19}$$

To analyse the average number of bits in (19), we may define A as in [24]. We calculate the term A as follows

$$\begin{aligned}
A &= \int_0^R \int_0^{2\pi} p\Big(\mathrm{SNR}_{e2,j}^{sc} > \gamma_e^2/r_j\Big) f\Big(r_j, \theta_j\Big) \mathrm{d}r_j \mathrm{d}\theta_j \\
&= \int_0^R \int_0^{2\pi} p\Big(\mathrm{SNR}_{e2,j}^{sc} > \gamma_e^2/r_j\Big) \frac{r_j}{\pi R^2} \mathrm{d}r_j \mathrm{d}\theta_j \\
&= \frac{2}{nR^2} \left(\frac{aP_d}{N_0 \gamma_e^2}\right)^{2/n} \Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_e^2}{aP_d}\right)
\end{aligned} \tag{20}$$

We define the term $B_i(S_{NL})$ as follows

$$\begin{aligned}
B_i(S_{NL}) = \int \ldots \int & p\big(\zeta_i > \gamma_e^1, \mathrm{SNR}_{e1,i}^{sc} < \gamma_e^1, \\
& \mathrm{SNR}_{i,b}^{sc} > \gamma_b/r_i, \theta_i, S_{NL}\big) f\big(r_i, \theta_i\big) \\
& \times \prod_{j \in S_{NL}} p\big(\gamma_e^1 < \mathrm{SNR}_{e2,k}^{sc} < \gamma_e^2/r_k\big) \\
& f\Big(r_j, \theta_j\Big) \mathrm{d}r_1 \mathrm{d}\theta_1 \ldots \mathrm{d}r_{NL} \mathrm{d}\theta_{NL} \mathrm{d}r_j \mathrm{d}\theta_j
\end{aligned} \tag{21}$$

Finally, we define the term D as follows

$$\begin{aligned}
D &= \int_0^R \int_0^{2\pi} p\big(\mathrm{SNR}_{e1,l}^{sc} < \gamma_e^1/r_l\big) f\big(r_l, \theta_l\big) \mathrm{d}r_l \mathrm{d}\theta_l \\
&= \int_0^R \int_0^{2\pi} p\big(\mathrm{SNR}_{e1,l}^{sc} < \gamma_e^1/r_l\big) \frac{r_l}{\pi R^2} \mathrm{d}r_l \mathrm{d}\theta_l \\
&= 1 - \frac{2}{nR^2} \left(\frac{aP_d}{N_0 \gamma_e^1}\right)^{2/n} \Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_e^1}{aP_d}\right)
\end{aligned} \tag{22}$$

It is clear that terms A and D are not dependent on the decoding result in the first phase and they are the same for all subscribers in the sets S_{NH} and S_{M-N}, respectively. Therefore, (19) can be written as (see (23))

For a given decoding result in the first phase, $B_i(S_{NL})$ is the same for all subscribers $i \in S_{M-N}$. In addition, for a fixed number of subscribers that decode the first enhancement layer correctly in the first phase, there are $\binom{M}{N}$ possible decoding results in which $|S_N| = N$. Similarly, given N

$$\begin{aligned}
b_2^r &= \frac{T_2 R_{E1}}{M} \sum_{N=1}^{M-1} \sum_{|S_N|=N} \sum_{N_L=1}^{N} \sum_{|S_{NL}|=N_L} \sum_{i \in S_{M-N}} \prod_{\substack{l \in M-N \\ l \neq i}} D \times \prod A \times B_i(S_{NL}) \\
&= \frac{T_2 R_{E1}}{M} \sum_{N=1}^{M-1} \sum_{|S_N|=N} \sum_{N_L=1}^{N} \sum_{|S_{NL}|=N_L} \sum_{i \in S_{M-N}} D^{(M-N-1)} \times A^{(N-N_L)} \times B_i(S_{NL})
\end{aligned} \tag{23}$$

doi: 10.1049/iet-com.2013.0167

subscribers decoded the first enhancement layer correctly in the first phase, there are $\binom{N}{N_L}$ possible decoding results in which $|S_{NL}| = N_L$. $B_i(S_{NL}) = B(N_L)$ is the same for all decoding combinations with number of relays equals to N_L. Based on the above analysis b_2^r in (9) can be written as

$$b_2^r = \frac{T_2 R_{E1}}{M} \sum_{N=1}^{M-1} \binom{M}{N} D^{(M-N-1)} \sum_{N_L=1}^{N} \binom{N}{N_L} (M-N) A^{(N-N_L)} B(N_L) \quad (24)$$

For a given subscriber, $j \in S_{NH}$ can eliminate the effect of the interference on the second enhancement layer using prior knowledge of the first enhancement layer message and perfect channel estimation. Accordingly, the effective SNR for the second enhancement layer is given by $\text{SNR}_{e,j} = \left(|h_j|^2 a P_d d_j^{-n} / N_0\right)$ which follows exponential distribution with a parameter $\lambda = \left(N_0 d_j^n / a P_d\right)$. Therefore, average total number of bits received by subscribers in the set S_{NH} in the second phase is given by

$$\begin{aligned} b_2^{BS} &= \frac{\int \ldots \int \sum_{i=1}^{M} T_2 R_{E2} P\left(\text{SNR}_{e2,i}^{sc} > \gamma_e^2 / r_i\right) f(r_i) \mathrm{d}r_i}{M} \\ &= \int_0^R T_2 R_{E2} P\left(\text{SNR}_{e2,i}^{sc} > \gamma_e^2 / r_i\right) f(r_i) \mathrm{d}r_i \\ &= \frac{2 T_2 R_{E2}}{n R^2} \left(\frac{a P_d}{\gamma_e^2 N_0}\right)^{(2/n)} \Gamma\left(\frac{2}{n}, \frac{R^n N_0 \gamma_e^2}{P_d}\right) \end{aligned} \quad (25)$$

According to (24) and (25), total average number of bits received in the second phase is given by

$$b^2 = b_2^r + b_2^{BS} \quad (26)$$

Average total multicast group throughput is given by

$$R = \frac{b^1 + b^2}{T_1 + T_2} \quad (27)$$

4.4 Average multicast throughput general equation

In this subsection, we are going to generalise (27) to hold all possible schemes described in this work. Equation (27) can be written as follows

$$\begin{aligned} R \times (T_1 + T_2) &= T_1 R_{B1} p\left(\text{SNR}_b > 2^{R_{B1}} - 1\right) \\ &+ T_1 R_{E1} p\left(\text{SNR}_e > 2^{R_{E1}} - 1 / \text{SNR}_b > 2^{R_{B1}} - 1\right) \\ &+ T_2 R_{E1} \sum_{N=1}^{M-1} \binom{M}{N} \left(p\left(\text{SNR}_e < 2^{R_{E1}} - 1\right)\right)^{M-N-1} \\ &\times \sum_{N_L=1}^{N} \binom{M}{N} \frac{M-N}{M} \\ &\left(p\left(\text{SNR}_e > 2^{R_{E2}} - 1 / \text{SNR}_b > 2^{R_{B1}} - 1\right)\right)^{N-N_L} \\ &\times p\left(\zeta > 2^{R_{E1}} - 1\right) \prod_{j \in S_{NL}} p \\ &\left(2^{R_{E1}} - 1 < \text{SNR}_e < 2^{R_{E2}} - 1 / \text{SNR}_b > 2^{R_{B1}} - 1\right) \\ &+ T_2 R_{E2} p\left(\text{SNR}_e > 2^{R_{E2}} - 1 / \text{SNR}_b > 2^{R_{B1}} - 1\right) \end{aligned} \quad (28)$$

where SNR_b and SNR_e are the effective received SNR for base and enhancement message layer, respectively. R_{B1} is the base message rate, R_{E1} and R_{E2} are the first and second enhancement layer message rates, respectively. Equation (28) is considered as a general equation to express the average multicast group throughput for different schemes by setting the parameters as illustrated in Table 1.

5 Simulation results

In this section, simulations are conducted to demonstrate the performance of the proposed scheme. We consider a circular cell of radius $R = 100$ m with the BS located in the centre of the cell. The multicast group consists of 30 subscribers, randomly distributed in the cell according to the uniform probability density function of r and θ. For a given subscriber, the joint probability density function of the distance between the subscriber and the BS r and the angle θ is given by $f(r, \theta) = r^2/\pi R^2$ with $0 \leq r \leq R$ and $0 \leq \theta \leq 2\pi$. The marginal distribution of r is given by $f(r) = 2r/R^2$ and θ is uniformly distributed over the range $[0, 2\pi]$ [24]. We assume that the sum of the transmitted powers by the relays is equal to the transmitted power by the BS. Other simulation parameters are listed in Table 2.

Table 2 Simulation parameters

Parameter	Value
P_d/N_0	85 dB
N	4
A	0.3
cell radius (R)	100

Table 1 Parameters ranges in each scheme

Scheme	T_1	T_2	A	R_b	R_{E1}	R_{E2}	N_L
conservative	T	0	0	0	0	0	0
superposition	T	0	$0 < a < 1$	$0 < R_b < \infty$	$0 < R_{E1} < \infty$	0	0
cooperative	$0 < T_1 < T$	$1 - T_1$	0	0	$0 < R_{E1} < \infty$	0	N
cooperative + superposition	$0 < T_1 < T$	$1 - T_1$	$0 < a < 1$	$0 < R_b < \infty$	$0 < R_{E1} < \infty$	$0 < R_{E2} < \infty$	$0 < N_L < N$

The generic mathematical expression for average network throughput derived in Section 4 can be applied on different channel models to optimise system parameters such as base rate, enhancement rates and power allocation factor. We applied the expression on Rayleigh fading channel to obtain a closed form of the average network throughput in (10). The closed form can be used in the design of system models to manipulate system parameters and the exact response of the system. This also gives a standard and unique solution in comparing different system protocols.

Fig. 4 compares the analytical model for the average multicast throughput with simulation results. We use Monte-Carlo simulations to calculate the values of $B(N_L)$. Based on the analytical expression, we present two studies for the effect of varying power allocation factor a and the percentage of subscribers that correctly decodes the enhancement layer in the first phase C.

Fig. 5 shows the analytical results of the average network throughput against transmitted power for different values of a, which matches exactly simulation results. The case $a = 0$ represents the conservative scheme and the maximum value of a can be calculated from (10). For high values of a, less number of subscribers will be able to correctly decode the base message. However, these subscribers will have higher probability to decode the two enhancement layers correctly since more power are allocated to these layers. In this case, the system serves relatively less number of subscribers with high data rate and blocks other subscribers. As a increases, the number of subscribers served will increase, but this will affect the rate of the enhancement layers resulting in relatively low average network throughput. Depending on a target block ratio and average network throughput, a can be easily chosen to be used in system design.

Fig. 6 demonstrates the effect of C on the average group throughput for different values of α. For higher values of α, the average throughput is lower because most of the subscribers cooperate in transmission and do not receive new data in the second phase. As α decreases, the average throughput increases as more subscribers receive the second enhancement layer. This hold true until most of the subscribers that decode correctly in the first phase can receive data in the second phase, and then the behaviour reverses. In other words, a small number of subscribers cooperate to transmit, which affects the throughput of the rest of the subscribers in the cell, resulting in decrease of the average throughput. As shown in the figure, $\alpha = 0.5$ is almost always better than $\alpha = 0.2$ or 0.8. This curve shows the existence of an optimal value of α, that will change depending on the rest of system parameters.

Fig. 7 demonstrates the effects of α on every subscriber throughput for a fixed C. Subscribers are sorted in a descending order according to their received SNR. For a fixed coverage ratio $C = 50\%$, when α increases, subscribers with low received SNR achieved higher throughput. This is because the number of subscribers that cooperate in transmission in the second phase increases. When α decreases, the throughput of subscribers with low SNR is affected but more subscribers are able to achieve higher throughput due to receiving data in the second phase.

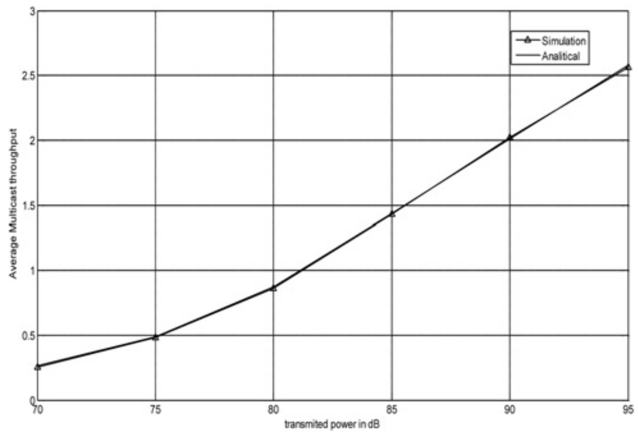

Fig. 4 *Average subscriber throughput for different values of P_d/N_0%*

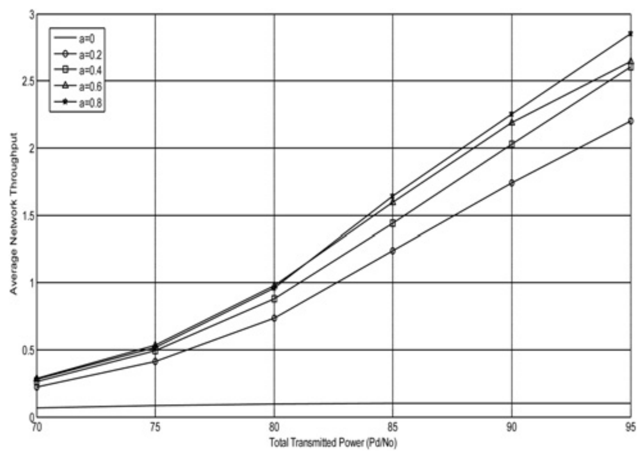

Fig. 5 *Average multicast group throughput for different values of a*

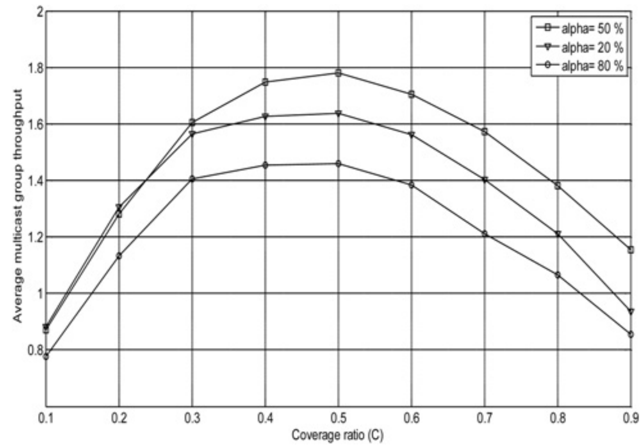

Fig. 6 *Average multicast group throughput for different values of C*

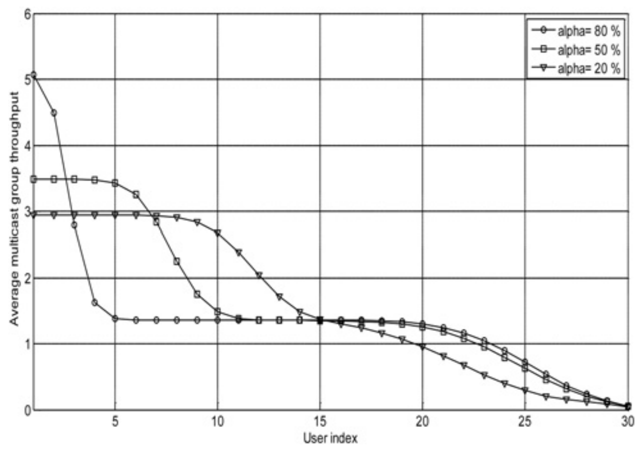

Fig. 7 *Average subscriber throughput for different values of α and C = 50%*

IET Commun., 2014, Vol. 8, Iss. 3, pp. 267–277
doi: 10.1049/iet-com.2013.0167

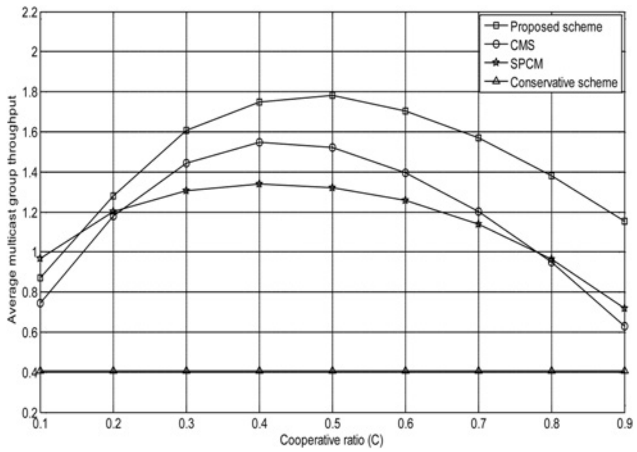

Fig. 8 *Average multicast group throughput for different values of C*

Fig. 8 compares the average throughput of the proposed scheme with the conservative scheme, the CMS proposed in [15] and SPCM proposed in [7]. CMS is based on distributive cooperative multicasting on two phases. For CMS, all subscribers that correctly decode the message in the first phase relay the message in the second phase. C is defined for CMS as the percentage of subscribers that decodes correctly in the first phase. SPCM utilises superposition where a multicast signal is generated by superimposing the base quality layer bit stream, modulated with a low-order modulation scheme to the enhancement quality layer [7]. The conservative scheme gives the lowest average throughput. The proposed scheme gives a better performance than conservative, CMS and SPCM for $C=$ 50%. In addition, the proposed scheme has more flexibility in controlling the performance of the subscribers with high SNR or the subscribers with low SNR and enhances the ability of scalable multimedia service delivery.

6 Conclusion

In this paper, a high-throughput CSM based on superposition coding is proposed. Such a scheme gives more degrees of freedom to drive the system either to higher data rates for subscribers with good channel conditions or to enhance the rate of subscribers with bad channel conditions. The scheme enhances the ability of scalable delivery of multimedia data by providing different rates for different subscribers resulting in different quality of service achieved among subscribers. We introduced a closed-form formulation of the average multicast group throughput that can be generalised to be applied for previous schemes stated in this paper. Simulations show that, under the same total energy consumption, the proposed scheme gives higher data rate for the multicast group than the conservative scheme and schemes exploiting cooperative relaying or superposition only.

7 Acknowledgments

This work was supported by a grant from Science and Technology Development Fund (STDF), Egypt.

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