Wideband Compressed Sensing Using Wavelet Packet Adaptive Reduced-set Matching Pursuit

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***Abstract*—One of the unsolved challenges in cognitive radio networks (CRNs) is the inability to sense a wideband spectrum in real-time. Traditional techniques require the use of analog-to-digital converters (ADCs) of very high sampling rate, given by the Nyquist theorem. Recently, compressed sensing has presented itself as an efficient solution for spectrum sensing aiming to reduce such requirement. However, the complexity and speed of traditional compressed sensing recovery algorithms not particularly developed for CRNs prevented such an application. In this paper, we present the Wavelet Packet Adaptive Reduced-set Matching Pursuit (WP-ARMP) approach for compressed wideband spectrum sensing. WP-ARMP is a fast and accurate greedy recovery algorithm for compressed sensing, which is suitable for real-time CRN applications. Furthermore, we exploit the sparsity of the spectrum in the wavelet packet domain. Simulation results show that our technique can reconstruct spectrum signals from samples collected at 1/4 the Nyquist sampling rate. The proposed scheme is not only much faster than other related techniques, but also results in over 99% probability of detection and a probability of false alarm below 1%.**

***Index Terms*—Cognitive Radio Networks; Spectrum Sensing; Compressed Sensing; Matching Pursuit**

I. Introduction

In order to allow Cognitive Radio Networks (CRNs) to efficiently utilize the spectrum, it is necessary to perform spectrum sensing over a wideband spectrum, in the order of several GHz. Unfortunately, existing hardware technologies are incapable of fulfilling such a requirement without scarifying the sensing time or accuracy.

Compressed sensing [1]–[3] is a recently developed sampling technique that senses many types of signals at a rate that is much lower than the Nyquist rate. Therefore, it was applied to spectrum sensing aiming to alleviate the high sampling rate requirements as in traditional techniques. Compressed sensing is applicable to either sparse signals or compressible signals, which only have a few significant coefficients in a suitable basis (e.g. Fourier, wavelets, …, etc.). Examples of such signals include natural images, videos, magnetic resonance imaging (MRI), radar, and radio spectrum signals [4]. The original signal can be recovered either by convex optimization or greedy recovery algorithms.

While ℓ_1 minimization provides an optimal solution for compressed sensing recovery, it is computationally expensive and not suitable for most real-time applications. Therefore, several greedy recovery algorithms have been developed for signal reconstruction [5]–[7]. Greedy recovery algorithms attempt to iteratively identify the support of the signal (its nonzero indices). This is done by correlating it with the sensing matrix columns and selecting the top magnitude values. Although such algorithms result in faster reconstruction time than ℓ_1 minimization, the reconstruction time and accuracy are still not suitable for real-time CRN applications.

In this paper, we present the Wavelet Packet Adaptive Reduced-set Matching Pursuit (WP-ARMP) for compressed wideband spectrum sensing in CRNs. The WP-ARMP contrasts with the existing compressed spectrum sensing techniques, which are either based on the impractical ℓ_1 minimization, or use greedy algorithms that are not fast and accurate enough. The WP-ARMP approach applies our adaptive reduced-set matching pursuit proposed in [8], [9] aiming to perform reconstruction of random signals as fast and accurately as possible. While most existing recovery algorithms select values from the whole correlation vector, WP-ARMP performs the selection from a reduced set of such vector, therefore achieving a significant speedup in the reconstruction process. Furthermore, the number of selected values is adapted from an iteration to another, based on the distribution of the correlation values. This contrasts with most of the existing algorithms which select a fixed number of elements per iteration, which usually results in selecting either too many or too few elements. After performing selection and support merging, WP-ARMP estimates the signal based on the identified support, and prunes it to exclude incorrectly selected elements.

Unlike most existing compressed wideband sensing techniques which exploit the sparsity of the spectrum in the frequency domain, WP-ARMP exploits the sparsity of the spectrum in the wavelet packet domain. This is due to the fact that the wavelet packet domain is capable of selecting the basis in which the signal is sparsest [10], thus improving performance. Combining ARMP with the wavelet packet domain, we introduce a fast and accurate spectrum sensing technique, the WP-ARMP, which is suitable for application in CRN.

The rest of this paper is organized as follows. Section II presents related work and reviews the fundamentals of compressed sensing. Section III presents the system model and the proposed technique. We evaluate the performance of our proposed technique in Section IV. Section V concludes the paper.

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II. Related Work and Background

A. Related Work

Various narrowband spectrum sensing techniques are widely known in the literature including matched filtering, energy detection, and cyclostationary feature detection [11]. Other techniques were also proposed to enable wideband spectrum sensing. Examples include multi-band joint detection [12] and wavelet-based spectrum sensing [13]. However, such techniques require ADCs that operate at a very high sampling rate, given by the Nyquist theorem, which is either too expensive or impossible to implement with contemporary hardware technologies. To reduce such high sampling rate requirement, some techniques employ several bandpass filters [14], transferring the problem of wideband spectrum sensing into multiple problems of narrowband spectrum sensing. However, this comes at the expense of excessive hardware requirements. Other techniques employ a tunable bandpass filter that sequentially performs spectrum sensing by sweeping the frequency range [15]. However, this comes at the expense of a larger sensing time, which is not suitable for most CRN applications.

Alternatively, [16] were the first to propose the use of compressed sensing to solve the spectrum sensing problem in CRNs. The spectrum signal is reconstructed using the TOMP algorithm [7]. Then, the boundaries between spectrum bands are estimated using a wavelet-based edge detector [13]. The Power Spectral Density (PSD) within each band is estimated. A two-step compressed sensing scheme is proposed in [17], which first obtains the signal sparsity, and then adjusts the number of collected samples. Alternatively, [18] adaptively adjusts the number of measurements without obtaining sparsity estimation in advance. In these two algorithms, the sparse domain utilized is the frequency domain, while the recovery algorithm is the ℓ_1 minimization. Cyclic feature detection is used in [19] in which the cyclic spectrum is obtained from the time-varying cross-correlation of the compressed measurements. The used recovery method was the optimal ℓ_1 minimization. In [20] ℓ_1 minimization was also used for signal reconstruction. However, the sensed spectrum was first transferred to a more spare domain using the Fourier-Haar wavelet packet domain. Then the same techniques were used for boundary and PSD estimation as in [16].

We use [16] and [20] techniques as our benchmarks for performance evaluation of our proposed approach as they represent the more accurate and the fastest existing techniques.

B. Compressed Sensing Fundamentals

Consider a sparse signal $x \in \mathbb{R}^n$, of sparsity level k. A measurement system that samples this signal to acquire m linear measurements is modeled as:

$$y = \Phi x, \tag{1}$$

where $\Phi \in \mathbb{R}^{m\times n}$ is the sensing matrix, and $y \in \mathbb{R}^m$ is the measurement vector.

Alternatively, the signal x may not be itself sparse, but it may be sparse in a certain basis Ψ, i.e. $x = \Psi s$, where s is a sparse vector. In this case, (1) is rewritten as:

$$y = \Phi\Psi s = As, \tag{2}$$

where Ψ is an $n \times n$ matrix whose columns form a basis in which x is sparse, and $A = \Phi\Psi$ is an $m \times n$ matrix, where $m \ll n$.

It was shown that the original signal x can be recovered from the measurement vector y, provided that the sensing matrix satisfies the Restricted Isometry Property (RIP) [1], [2]. Random matrices whose elements follow Gaussian, Bernoulli or sub-Gaussian distributions satisfy the RIP with high probability [21].

For signal recovery, ℓ_1 minimization was originally suggested as follows [22]:

$$\hat{x} = \arg\min_z \|z\|_1 \text{ subject to } y = \Phi z \tag{3}$$

While ℓ_1 minimization is a powerful solution for the sparse problem, it is computationally expensive [1].

III. WP-ARMP Wideband Spectrum Sensing

In this section, we present the Wavelet Packet Adaptive Reduced-set Matching Pursuit (WP-ARMP) approach for compressed spectrum sensing. WP-ARMP applies ARMP to the wideband spectrum in the wavelet packet domain. WP-ARMP is a greedy compressed sensing reconstruction algorithm. WP-ARMP targets the selection of an optimum number of elements of the correlation vector per iteration, which results in a high reconstruction accuracy at a low complexity. Not only does such technique result in a significant reduction in the required sampling rate, but it is also capable of fast and accurate spectrum sensing, which is suitable for real-time CRN applications.

A. System Model and Problem Statement

We consider the generic spectrum sensing model adopted in [13], [16], [20]. A wideband wireless radio environment, in which multiple primary users (PUs) and secondary users (SUs) coexist, is assumed. The spectrum to be sensed is divided into N non-overlapping bands in the range of [f_0 to f_N] Hz, each of equal bandwidth. In case the bandwidths are not equal or unknown by the SUs, an extra step for boundary detection is required, which can be simply performed using a wavelet-based edge detector. The bands are denoted by B_n, and their boundaries are denoted by $f_0 < f_1 < \ldots < f_N$. Only a number K out of the total number of bands, N, are occupied by PUs, while the rest of the bands are free. The SUs sense the spectrum aiming to find the free bands. The SUs need to identify the PSD of the N bands. The signal received by a SU is given by

$$r(t) = \sum_{n=1}^{N} h_n p_n(t) + w(t) \tag{4}$$

and is of PSD given by [23]

$$S_r(f) = \lim_{T\to\infty} E\left[\frac{1}{T}\left|\int_0^T r(t)e^{-j2\pi ft}dt\right|^2\right] \quad (5)$$

which leads to

$$S_r(f) = \sum_{n=1}^{N} h_n^2 S_n(f) + S_w(f), f \in [f_0, f_N] \quad (6)$$

where h_n^2 is the PSD in the n^{th} band, $w(t)$ is additive white Gaussian noise with PSD $S_w(f)$, and $p_n(t)$ is a time-domain signal whose PSD is

$$S_n(f) = \begin{cases} 1 & f \in B_n \\ 0 & f \notin B_n \end{cases} \quad (7)$$

The spectrum sensing problem is stated as follows: *Given the received signal $r(t)$ at a SU, it is required to estimate the PSD in each band $\{h_n^2\}_{n=1}^N$ as fast and accurately as possible.*

B. Wavelet Packet Spectrum Sensing

Let the vector $r_t \in \mathbb{R}^{\mathrm{M}}$ denotes the samples of the received signal $r(t)$ collected at the Nyquist rate. The vector $x_t \in \mathbb{C}^{\mathrm{K}}$ that represents the samples that are collected sparsely and randomly from r_t can be represented as:

$$x_t = Sr_t \quad (8)$$

where S is a $K \times M$ projection (or sampling) matrix. If S is taken to be the unit matrix, Nyquist rate sampling is performed. However, the target is to perform sub-Nyquist sampling. Therefore, we will have $K < M$.

The sample vector r_t obtained at the Nyquist rate corresponds to the vector r_f in the frequency domain. We seek to obtain r_f, which represents the samples of the spectrum, which can be expressed as:

$$r_f = F_M r_t \quad (9)$$

where F_M is the M-point Discrete Fourier Transform (DFT) matrix. Substituting in (8), we obtain:

$$x_t = SF_M^{-1} r_f \quad (10)$$

Denote the Haar wavelet packet matrix by W_p. Thus, we have

$$s = W_p r_f \quad (11)$$

and

$$r_f = W_p^{-1} s \quad (12)$$

where s is the wavelet transform of r_f. Using compressed sensing techniques, the spectrum can be estimated as follows:

$$\hat{r_f} = \arg\min_{r_f} \|r_f\|_1, \quad s.t. Ar_f = x_t \quad (13)$$

where $A = SF_M^{-1}W_p^{-1}$. The PSD within each band is then estimated by averaging the samples in each band.

C. Adaptive Reduced-set Matching Pursuit

The adaptive reduced-set matching pursuit greedy recovery algorithm is capable of spectrum reconstruction with high accuracy at a low complexity. The main idea is the improved selection strategy, in which the element selection process is adaptive. Furthermore, the selection is performed from a reduced set of the correlation values. This contrasts with most of the related existing algorithms in which the selection is performed from the whole correlation vector and is performed in a non-adaptive manner.

Initially, the spectrum estimate is set to zero and the residual to the measurement vector. In each iteration, the following steps are performed:

1) **Correlation:** The support of the sparse wavelet packet spectrum signal is iteratively identified. This is done by correlating the residual with the sensing matrix columns obtaining a vector g.
2) **Selection and Support Merging:** First, a reduced set of correlation values is formed by taking the βk largest magnitude elements. Then, those elements of magnitudes larger than or equal to a fraction $0 < \alpha < 1$ of the top magnitude value are selected. The indices of the selected elements are merged with the already identified support set.
3) **Signal estimation:** An estimate of the spectrum signal is formed by least square minimization. This is done via multiplication by the pseudo-inverse of the sensing matrix.
4) **Pruning:** The estimated spectrum signal is pruned by only keeping the k largest magnitude components and setting the rest to zero. This removes the incorrectly selected elements from the support set, and prevents them from degrading the performance in the subsequent iterations.
5) **Residual Calculation:** The new residual is calculated by subtracting the contribution of the estimated signal from the measurement vector. This residual is then correlated with the sensing matrix columns for the successive iterations.

Algorithm 1 summarizes how the ARMP algorithm is applied to the spectrum sensing problem. The WP-ARMP algorithm steps are repeated until a stopping criterion is met. The operator $L_k(\cdot)$ returns the index set of the k largest magnitude elements of its argument vector. The hard thresholding operator $H_k(\cdot)$ retains only the k elements with the largest magnitudes and sets the rest to zero. A_T is a matrix that contains the columns of A at indices from the set T.

The values of α and β affect the performance of the proposed approach. Too high and too low values of α and β slow down the performance and reduce the reconstruction accuracy as the case with related works which perform selection from the entire set. However, we have shown in [9] that the performance of the algorithm is not sensitive to the α and β values as long as they are in a moderate region of $\alpha \in [0.5, 0.7]$ and $\beta \in [0.15, 0.75]$. This is due to the fact

Algorithm 1 WP-ARMP Algorithm

Input: Matrix $A = SF_M^{-1}W_p^{-1}$, sparsely collected samples x_t, sparsity level k, parameters α and β.

Initialize: $\hat{r_f}^{[0]} = 0,\ residual^{[0]} = x_t,\ T^{[0]} = \emptyset$.

for $i = 1; i := i + 1$ until the stopping criterion is met **do**

$g^{[i]} \leftarrow A^* \times resdiual^{[i-1]}$ {Correlation vector}

$J \leftarrow L_{\beta k}(g^{[i]})$ {Indices of the βk largest magnitude elements in g}

$W \leftarrow \{j : |g_j^{[i]}| \geq \alpha \max_l |g_l^{[i]}|, j \in J\}$ {Indices of elements in J of magnitudes larger than or equal to $\alpha \max_l |g_l^{[i]}|$}

$T \leftarrow W \cup \mathrm{supp}(\hat{r_f}^{[i-1]})$ {Support merging}

$b|_T \leftarrow A_T^{\dagger} x_t,\ b|_{T^c} \leftarrow 0$ {Signal estimation}

$\hat{r_f}^{[i]} \leftarrow H_k(b)$ {Prune approximation}

$resdiual \leftarrow x_t - A\hat{r_f}^{[i]}$ {Update residual}

end for

Output: Reconstructed spectrum samples $\hat{r_f}$

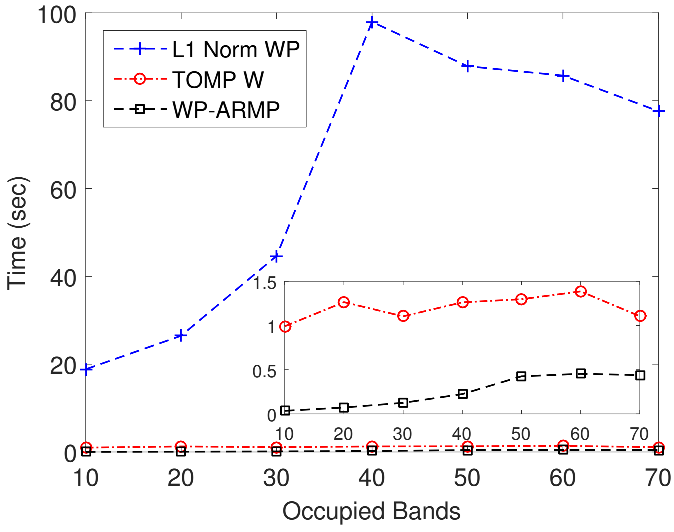

Figure 1. Reconstruction time (sec) for different number of occupied channels.

that the number of selected elements per iteration becomes neither too large nor too small in this region. This in turn results in a high reconstruction accuracy at a low complexity. The selection of α and β is discussed in detail in [9].

IV. Performance Evaluation

A. Simulation Setup

In this section, we compare the performance of WP-ARMP for compressed wideband spectrum sensing, against ℓ_1 minimization in the wavelet packet domain [20] and TOMP [16]. Since our test signals are spectrum signals (in the frequency domain), the application of $\ell_1$minimization in the wavelet packet domain corresponds to the Fourier-Haar wavelet packet domain proposed in [20]. WP-ARMP is utilized in the Haar wavelet packet domain. 4-level Haar wavelet packet decomposition is performed. The best basis algorithm [10] is used to select the best basis in which the spectrum has the sparsest representation.

For each algorithm, the reported results are the average of the metrics evaluated for 50 independent trials. In each trial, we generate a random sparse signal of length $n = 1000$, corresponding to the Nyquist-rate spectrum samples. The total bandwidth is divided into 100 equal channels, each consisting of 10 samples. For TOMP, we take the relaxing coefficient $\alpha = 0.9$ and the downward extending coefficient $l = 2$ levels [7]. For ARMP, we take $\alpha = 0.7$ and $\beta = 0.25$ [9].

B. Performance Metrics

The performance metrics that we use to compare our WP-ARMP algorithm against other related algorithms are as follows:

- The reconstruction time in seconds, the time required by the algorithm to reconstruct the spectrum signal from the measurement signal.
- Probability of detection, the percentage of occupied bands detected by the algorithm.
- Probability of false alarm, the percentage of free bands erroneously classified by the algorithm as occupied bands.

C. Simulation Results

1) Performance against number of occupied bands: We compare the algorithms against the number of occupied bands. The number of occupied bands is varied from 10 to 70 at steps of 10, corresponding to spectrum occupancy from 10% to 70%, respectively. The amplitudes of the bands are uniformly distributed from 0 to 100. We take 400 measurements, corresponding to a compression ratio of 40%. In the following figures, W stands for wavelet domain and WP stands for wavelet packet domain.

Figure 1 depicts the reconstruction time required by the algorithms. As expected, ℓ_1 minimization takes the longest time, from about 20 to 100 seconds, according to the channel occupancy. Such excessive time is not appropriate for cognitive radio applications. Then, TOMP takes up to about 1.4 seconds. TOMP is slow due to the excessive number of projections performed by algorithm for selecting the best subtrees. WP-ARMP achieves the lowest time ranging from 0.035 to 0.45 seconds for low and high channel occupancies respectively. Such significant speedup is due to the adaptive selection of correlation values from a reduced set.

Figure 2 illustrates the probability of detection. For lower channel occupancy, ℓ_1 minimization and WP-ARMP give 100% probability of detection. In this range, TOMP gives lower detection of about 95%. For higher occupancy, the algorithms are very close at about 97%.

The probability of false alarm is shown in Figure 3. For lower occupancy up to about 40%, ℓ_1 minimization and ARMP give about 0% probability of false alarm. In this range, TOMP gives a significantly higher probability of false alarm. For higher occupancies, ℓ_1 minimization gives the lowest value, followed by WP-ARMP and TOMP. It should be noted that the lower probability false alarm of ℓ_1 minimization comes at

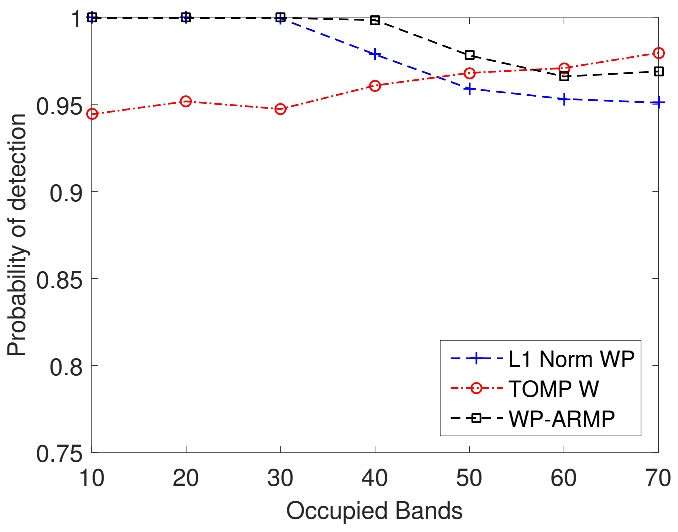

Figure 2. Probability of detection for different number of occupied channels.

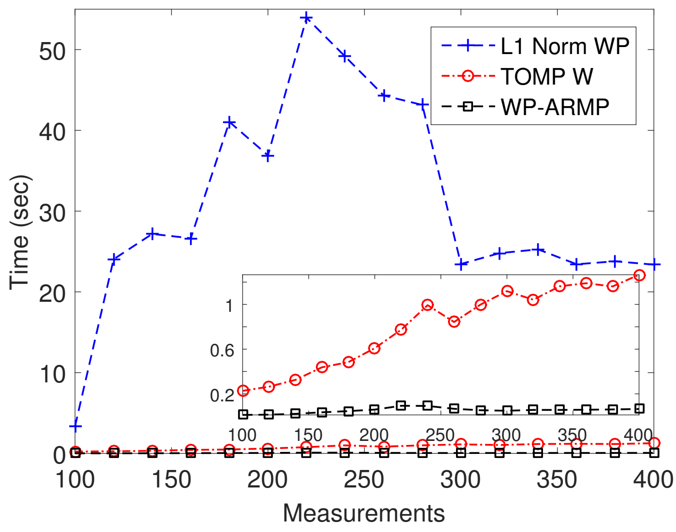

Figure 4. Reconstruction time (sec) for different number of measurements.

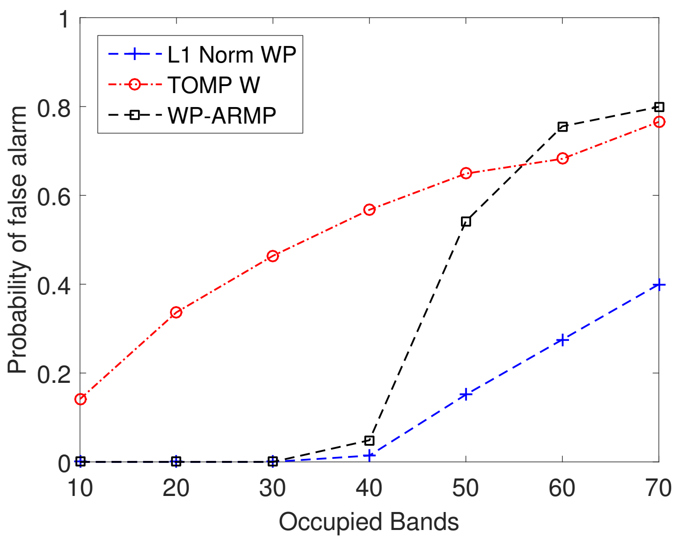

Figure 3. Probability of false alarm for different number of occupied channels.

the expense of excessively high reconstruction time, which is not appropriate for cognitive radio applications.

The previous results show that WP-ARMP achieves an enormous complexity reduction compared to ℓ_1 minimization with close reconstruction capability. WP-ARMP also achieves an enormous improvement in reconstruction accuracy compared to TOMP at a considerably lower complexity.

2) Performance against number of measurements: We next compare the performance of the algorithms against the number of measurements taken, which indicates the reduction in sampling rate achieved through the use of compressed sensing. The sparsity of the signal is 20% (i.e. it has 200 nonzero samples). This corresponds to a spectrum occupancy of 20%. The nonzero points are evenly divided into 20 channels, each consisting of 10 samples. The amplitudes of the channels are uniformly distributed from 0 to 100. To evaluate the benefit from compressed sensing, we use a fewer number of measurements than n. We plot the performance metrics against the number of measurements, ranging from 100 to 400 with steps of 20 samples, corresponding to compression ratios from 10% to 40%.

Figure 4 depicts the reconstruction time required by the algorithms. Again, ℓ_1 minimization takes the longest time, in the range of 20-55 seconds. Such excessive time is not appropriate for cognitive radio applications. Then, TOMP takes up to about 1.3 seconds. Next is our WP-ARMP algorithm, at a significantly lower time of about 0.09 seconds.

Figure 5 illustrates the probability of detection. Comparing the algorithms in the region where the probability of detection is almost 100% and probability of false alarm is about 0%, i.e. for a number of measurements of above 250, we see that WP-ARMP gives about 99.2% probability of detection at 260 and about 100% at 280 measurements. ℓ_1 minimization gives a lower value of about 96.2%. In the same region, TOMP gives a lower probability of detection of about 94%.

The probability of false alarm is shown in Figure 6. In the same region, ℓ_1 minimization and WP-ARMP give the lowest probability of false alarm at about 1%. TOMP shows a considerably higher probability of false alarm compared to the rest of the algorithms of about 35% to 55%. While ℓ_1 minimization gives slightly lower probability of false alarm, this comes at the expense of a lower probability of detection, which is dangerous, since it may result in interference with PUs.

The previous results show that using WP-ARMP, a compression ratio of about 25% can be achieved, with over 99% probability of detection and below 1% probability of false alarm. An enormous complexity reduction is achieved compared to ℓ_1 minimization (from more than 45 seconds to 0.09 seconds) at very close probability of false alarm and even higher probability of detection. Moreover, an enormous improvement in reconstruction accuracy is achieved compared

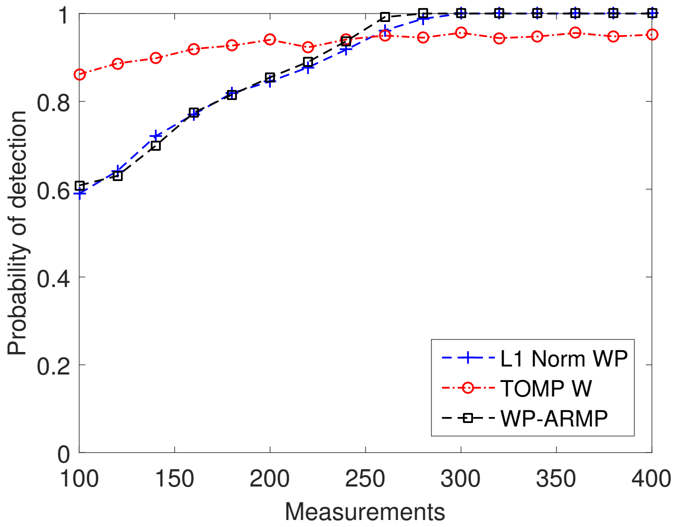

Figure 5. Probability of detection for different number of measurements.

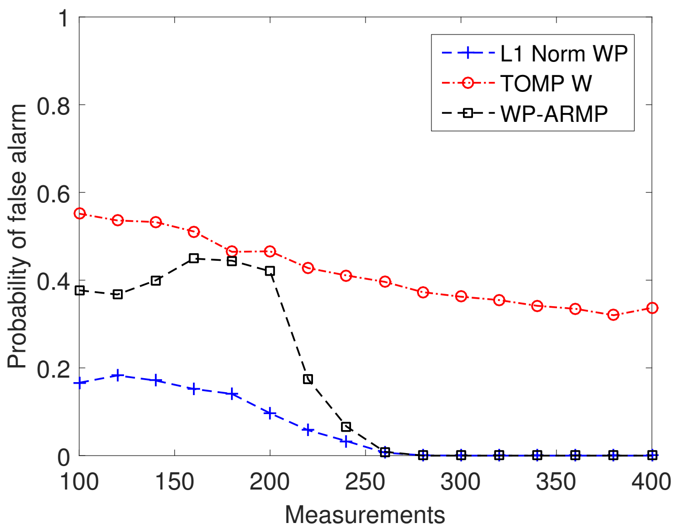

Figure 6. Probability of false alarm for different number of measurements.

to TOMP (from 95% to 99.2% probability of detection and from 40% to about 1% probability of false alarm) at a considerably lower complexity (from 0.85 to 0.09 seconds).

V. Conclusion

In this paper, we have presented a practical technique for wideband spectrum sensing in cognitive radio networks. The proposed WP-ARMP applies the adaptive reduced-set matching pursuit approach in Haar wavelet packet domain of the wideband sensed spectrum. WP-ARMP is a fast and accurate compressed sensing recovery algorithm that exploits the sparsity of the spectrum signals in the Haar wavelet packet domain. We have demonstrated that our technique is capable of reconstructing spectrum signals from samples collected at a rate of about 1/4 the Nyquist rate, significantly faster than other related algorithms. The probability of detection is over 99% and the probability of false alarm is below 1%.

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