Homewor 1 Solutions

Problem 1:

a) Since $N(R) = \sqrt{2\pi} f_m \rho e^{-\rho^2}$
the ratio of the desired signal level to the r.m.s. signal level that maxmizes $N(R)$ is the solution of

$$\frac{\partial N(\rho)}{\partial \rho} = 0 \Rightarrow \sqrt{2\pi} f_m (1 - 2\rho^2) e^{-\rho^2} = 0$$
$$\Rightarrow 1 - 2\rho^2 = 0 \Rightarrow \boxed{\rho^* = \frac{1}{\sqrt{2}}}$$

b) $$\lambda = c/f = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3} m$$

$$v = \frac{5 \times 1000}{60 \times 60} = 13.89 \ m/sec$$

$$f_m = \frac{v}{\lambda} = 41.67 \ Hz$$

Given $\rho = \frac{1}{\sqrt{2}}$ from part (a)

$$N(\rho) = \sqrt{2\pi} \cdot 41.67 \cdot \frac{e^{-\frac{1}{2}}}{\sqrt{2}} = 44.8 \quad \text{crossing/sec}$$

Therefore, the maximum # of crossing per minute
$= 44.8 \times 60 = 2688$ crossings

c) $$T_{fade}(R) = \frac{Pr[|r(t)| < R]}{N(R)} = \frac{1 - e^{-\rho^2}}{N(R)}$$
$$= \frac{1 - e^{-\frac{1}{2}}}{44.8} = 8.8 \ m.Sec$$

Problem 2:

$$\overline{T_{fade}}(R) = 1 \text{ m.sec} = \frac{P_r[|r(t)| < R]}{N(R)}$$

$$= \frac{1 - e^{-\rho^2}}{N(R)}$$

$$\rho = \frac{R}{R_{rms}}$$

$$\therefore \rho_{dB} = R_{dB} - R_{rms\,dB} \quad \ldots\ldots (1)$$

Give that $R_{dB} = R_{r.m.s} - 10 \text{ dB}$

sub in (1)

$$\rho_{dB} = \cancel{R_{r.m.s.}} - 10\text{ dB} - \cancel{R_{r.m.s.}\text{ dB}}$$

$$= -10 \text{ dB}$$

$$= 0.3162$$

$$\therefore N(R) = \sqrt{2\pi}\, f_{d,max}\, \rho e^{-\rho^2}$$

$$f_{d,max} = V/\lambda \quad , \quad \lambda = \frac{3 \times 10^8}{900 \times 10^6} = \frac{1}{3}$$

$$\therefore f_{d,max} = 3V \quad \text{Hz}$$

$$\therefore \overline{T_{fade}}(R) = 1 \text{ m. sec} = \frac{1 - e^{-(0.3162)^2}}{\sqrt{2\pi} \cdot 3V \cdot (0.3162) \cdot e^{-(0.3162)^2}}$$

$$\boxed{\therefore V = 44.22 \text{ m/s}}$$

and hence

$$f_{d,max} = 3 \times 44.22 = 132.68 \text{ Hz}$$

$$\& \; N(R) = 95.162 \quad \text{crossings/sec}$$

$\therefore$ # crossings (i.e. fades) in 10 sec

$$= 95.162 \times 10 = \underline{\underline{951.62}}$$

Problem (3):

a) mean excess delay

$$\bar{\tau} = \frac{1\times 0 + 0.1\times 1 + 1\times 2}{1 + 0.1 + 1} = \frac{2.1}{2.1} = 1\ (\mu s)$$

$$\overline{\tau^2} = \frac{1\times 0^2 + 0.1\times 1^2 + 1\times 2^2}{1 + 0.1 + 1} = 1.95\ (\mu s^2)$$

$\therefore$ r.m.s delay spread $= \sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$

$$= \sqrt{1.95^2 - 1^2}$$

$$= 0.976\ \mu s$$

b) maximum excess delay (20 dB) $= 2\ \mu s$

c) $T_{min} = 10\ \sigma_\tau = 10 \times 0.976 = 9.76\ \mu s$

$\therefore$ maximum RF Symbol Rate $= \dfrac{1}{T_{min}}$

$$= \frac{1}{9.76 \times 10^{-6}}$$

$$= 102\ \text{Kbp}$$

d) Assume $f_c = 900\ \text{MHz}$

$$\lambda = \frac{c}{f_c} = 0.33\ m$$

$$V = \frac{30 \times 1000}{60 \times 60} = 8.33\ m/s$$

$$f_m = \frac{V}{\lambda} = \frac{8.33}{0.33} = 25\ Hz$$

$\therefore$ Coherence Time $T_c = \dfrac{0.423}{25} = 0.017\ s$

since $T_c = \dfrac{0.423}{f_m}$

e) for B.W. = 30 KHz

Flat fading $\rightarrow T_s > T_{min}$, $T_{min} = 9.76\ \mu s$

$\Rightarrow T_s > 9.76\ \mu s$

so $T_s = \dfrac{1}{30K} = 33\ \mu s$

i.e. T_s greater than T_{min}

$\Rightarrow$ Channel will exhibit flat fading

for B.W. = 1MHz

$T_s = 1\ \mu s < T_{min}$

$\therefore$ This channel will exhibit frequency-selective fading