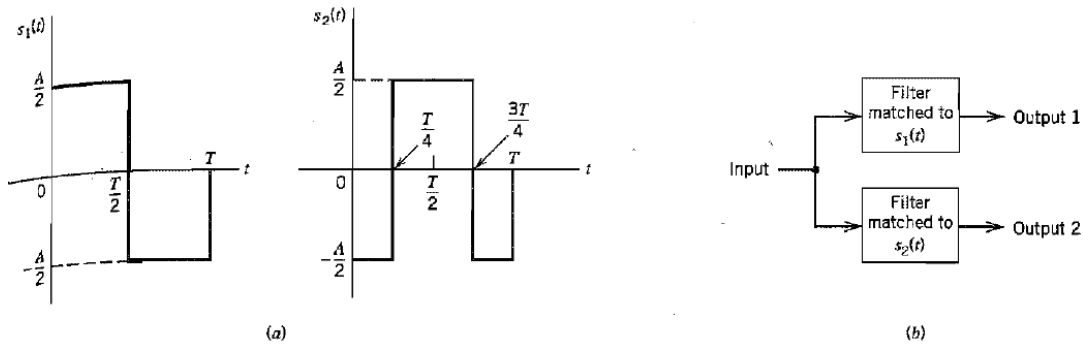# PROBLEMS SET I

REVISION

Problem (1-6) (Haykin, 5th edition) 5.15 (a, b, c), 5.16, 5.17, 5.20, 5.23, 5.24*

MATCHED FILTERS

Problem (1) (Haykin, 5th edition) 8.1

Problem (2) The figure below shows a pair of pulses that are orthogonal to each other over the interval [0, *T*]. In this problem, we investigate the use of this pulse-pair to study a two-dimensional matched filter.


(a)   (b)

a) Determine the matched filters for the pulses $s_1(t)$ and $s_2(t)$ considered individually.
b) Form a two-dimensional matched filter by connecting the two matched filters of part (a) in parallel, as shown in Fig. (b) above. Hence, demonstrate the following:
   i. When the pulse $s_1(t)$ is applied to this two-dimensional filer, the response of the lower matched filter is zero.
   ii. When the pulse $s_2(t)$ is applied to this two-dimensional filer, the response of the upper matched filter is zero.

Problem (3) In this problem, we explore a method for the approximate realization of a matched filter using the simple RC low pass filter shown below. The frequency response of the filter is given by:

$$H(f) = \frac{1}{1+jf/f_o} \ ,$$

where $f_o = \frac{1}{2\pi RC}$. The input signal $g(t)$ is a rectangular pulse of amplitude *A* and duration *T*. The requirement is to optimize the selection of the 3-dB cut off frequency f_o of the filter such that the peak pulse signal-to-noise ratio at the filter output is maximized. With this objective in mind, show that the optimum value of f_o is 0.2/*T*, for which the loss in the signal-to-noise ratio compared to the matched filter is about 1 dB.
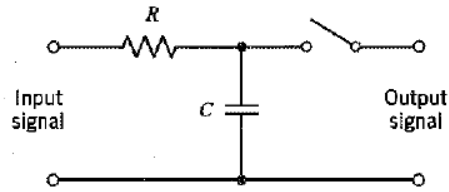
Problem (4*) Consider the signal

$$s(t) = \begin{cases} \left(\dfrac{A}{T}\right) t\,cos(2\pi f_c t), & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

  c) Determine the impulse response of the matched filter for the signal.
  d) Determine the output of the matched filter at $t = T$.
  e) Suppose the signal $s(t)$ is passed through a correlator that correlates the input $s(t)$ with (t) . Determine the value of the correlator output at $t = T$. Compare your result with that in (b).

PROBABILITY OF ERROR CALCULATION

Problem (1-3) (Haykin, 5th edition) 8.8, 8.5*, 8.6.

Problem (4*) Repeat problem 8.5 for the case of unipolar return-to-zero signaling, in which case symbol 1 is represented by a pulse of amplitude A and duration $T_b/2$ and symbol 0 is represented by transmitting no pulse.

Problem (5) In this problem, we revisit the PCM receiver discussed in the lecture, but this time we consider the use of bipolar NRZ signaling, in which case the transmitted singal $s(t)$ is defined by:
  Binary symbol 1: $s(t) = \pm A$ for $0 \le t \le T$
  Binary symbol 0: $s(t) = 0$ for $0 \le t \le T$
Determine the average probability of symbol error for this receiver assuming that the binary symbols 0 and 1 are equiprobable.

Problem (6-8) (Lathi, 3rd edition) 13.2-2, 13.2-3*, 13.5-1

- (*) Starred problems are HW problems.