

# MATLAB Simulation of Optimum Receiver for Detecting BPSK and BFSK Signals

Due Date: 13/5/2023

## 1 Introduction

As most of the communication channels are bandpass in nature, digital signals are modulated by a carrier wave of appropriate frequency before transmission. In order to distinguish the transmitted bits in the presence of noise with minimum probability of error, a receiver has to be carefully designed. For the detection of binary signals in spectrally white noise, the matched filter is optimum in the sense that the output signal-to-noise ratio (SNR) is maximized at the end of the bit period and thus the probability of error in the threshold detection is minimized.

In this project, we would like to design the optimal receiver for the detection of binary Frequency Shift Keying (FSK) and Phase Shift Keying (PSK) signals and investigate the performance in each case.

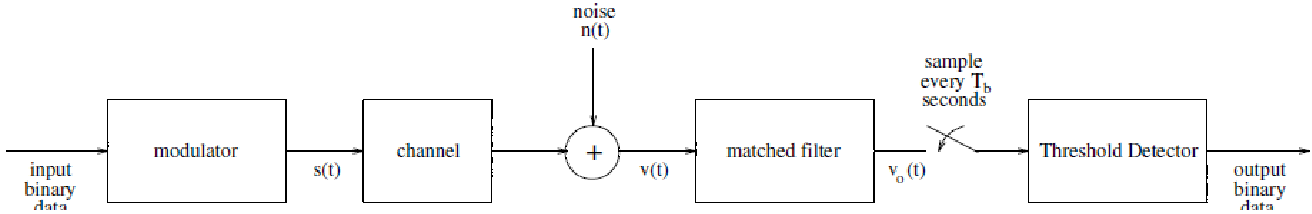


Figure 1: Block Diagram of a Binary Data Transmission System.

## 2 Experiment

Consider the binary transmission system as shown in Figure 1. The input to the system is a binary sequence  $\{b_k\}$  of bit duration  $T_b$ . Depending on the modulation scheme being employed, the modulated signal  $s(t)$  takes on different waveforms. For BFSK systems, the modulated signals are given by

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_1 t) & \text{if } b_k = 1 \\ s_2(t) = A\cos(\omega_2 t) & \text{if } b_k = 0 \end{cases} \quad (1)$$

where  $0 < t < T_b$ ,  $A$  is the amplitude of the modulated waveform, and

$$\omega_1 = \frac{2\pi(n_0 + n_1)}{T_b} \quad (2)$$

$$\omega_2 = \frac{2\pi(n_0 - n_1)}{T_b} \quad (3)$$

In this project, we choose  $n_0 = 4$  and  $n_1 = 1$ .

For BPSK modulation, we can write

$$s(t) = \begin{cases} s_1(t) = A\cos(\omega_c t) & \text{if } b_k = 1 \\ s_2(t) = -A\cos(\omega_c t) & \text{if } b_k = 0 \end{cases} \quad (4)$$

where  $0 < t < T_b$ ,  $A$  is the amplitude of the modulated waveform, and we choose

$$\omega_c = 4 \frac{2\pi}{T_b} \quad (5)$$

The modulated signal will then be transmitted through the channel, and is corrupted by some additive noise  $n(t)$ . For simplicity, the channel is assumed to be ideal and the channel noise is assumed to be white and Gaussian with power spectral density

$$S_n(\omega) = \frac{N_o}{2} \quad (6)$$

The optimum receiver consists of a matched filter, a sampler, and a threshold detector. The signal-plus-noise  $v(t)$  is filtered, producing  $v_o(t)$ . The transmitted bit is obtained by comparing the sampled  $v_o(t)$  at the end of the bit period with a pre-determined threshold  $V_T$  according to the following rule:

$$\hat{b}_k = \begin{cases} 1 & \text{if } v_o(T_b) > V_T \\ 0 & \text{if } v_o(T_b) < V_T \end{cases} \quad (7)$$

### 3 Procedure

In computer simulations, a very important issue is that the signals and noise have to be sampled. Since the bit period is  $T_b$ , and in the above theory, we assumed the transmitted sinusoidal pulses have rectangular envelope, therefore, to represent the continuous signal shapes adequately, we must sample at a substantially higher frequency than the bit frequency. Let the sampling frequency be

$$\omega_s = \frac{2\pi}{T_s} = N \cdot \frac{2\pi}{T_b} \quad (8)$$

Here in this project, we choose  $T_s$  to be 1 and  $N$  to be 40. Using this sampling period, we generate the following:

1. Generate a random sequence of binary bits  $\{b_k\}$  using MATLAB. Note that this sequence of binary digits are represented by rectangular pulses having amplitudes  $\pm 1$ , each lasting for a period of  $T_b$ . What is the effective bandwidth of this sequence of rectangular pulses?  
This sequence of pulses will modulate different carriers of amplitude  $A$  in the transmission either by FSK or by PSK as described in Section 2 and then be sampled at  $T_s$ .
2. Generate a zero-mean, white Gaussian random sequence  $w(nT_s)$  sampled at  $T_s$ . This noise sequence has a two-sided power spectral density of  $\frac{N_o}{2}$ . In this project, we fix  $N_o = 2$ . Note that since  $w(nT_s)$  is sampled, the power of this noise sequence depends on the sampling period  $T_s$ , the smaller is  $T_s$ , the larger is the noise power.
3. Since the noise sequence  $w(nT_s)$  has a bandwidth much larger than the effective bandwidth of the signal pulses, the noise outside the effective bandwidth is irrelevant to the transmission of the data. We therefore define an effective input signal-to-noise ratio  $\rho$  such that

$$\rho = \frac{A^2 T_b}{2N_o} \quad (9)$$

where  $A$  is the amplitude of the modulated pulse. When expressed in dB, we have

$$SNR_e = 10 \log_{10} \left[ \frac{A^2 T_b}{2N_o} \right] \quad (10)$$

In this project, we investigate the transmission and reception of FSK and PSK signals in an effective input signal-to-noise ratio range of -4 dB to 4 dB.

4. With a fixed value of  $N_o = 2$ , for each chosen value of  $SNR_e$  in the range of -4 dB to 4 dB, generate a sequence of FSK modulated pulses and another sequence of PSK modulated pulses using the value of  $A$  calculated. This procedure yields the transmitted signal  $s(t)$ .
5. The sampled noise sequence is added to each of the  $s(t)$  representing the FSK and PSK sequences. This mixture yields the noisy signal  $v(t)$  resulted from transmitting through the channel.
6. Each of the noisy signal sequences is then passed through the matched filter designed respectively for the specific transmission scheme. Denote the output of the matched-filter at the end of each bit period by  $v_o(kT_b)$ .
7. Compare  $v_o(kT_b)$  with the respective threshold in each of the cases and decide if the transmitted bit  $b_k$  is 1 or 0.
8. For a suitably selected number of bits transmitted in the sequence, evaluate the bit-error rate (experimental probability of error) of the each of the transmission schemes at the specific  $SNR_e$ . Bit-error rate is the number of erroneous bit at the output in comparison with the transmitted binary sequence, i.e.,

$$P_{exp} = \frac{\text{total number of erroneous bits}}{\text{total number of transmitted bits}} \quad (11)$$

9. Since noise is a random process, repeat Step 1 through Step 8 for 20 realizations of noise and calculate the average bit-error rate,  $P_{exp}$ , for the same chosen  $SNR_e$ .
10. Repeat Step 1 through Step 9 for another value of effective signal-to-noise ratio ( $SNR_e$  ranges from -4 dB to 4 dB).
11. Plot the performance graphs of binary FSK and PSK transmissions (Bit-Error Rate against Signal-to-Noise Ratio). Compare these curves with the theoretical performance and comment on your results.

## 4 Logistics

- You are not allowed to use MATLAB built in PSK and FSK functions. You have to write your own code.
- Each team is composed of 2 students.
- The report is no longer than 10 pages.
- The grade is mainly on the reported results and on your analysis of the results (in addition to the writing quality).
- Your code should be added as an appendix (not counted as part of the 10 pages of the report).
- The submission material (report + codes) should be emailed only by 1 team member to (akhattab@eng.cu.edu.eg) cc'ing the TA.
- Project Due: 13/5/2023.